

Lander-Aided Differential Doppler and One-Way Ranging for Lunar Rover Navigation with a Single Relay Satellite

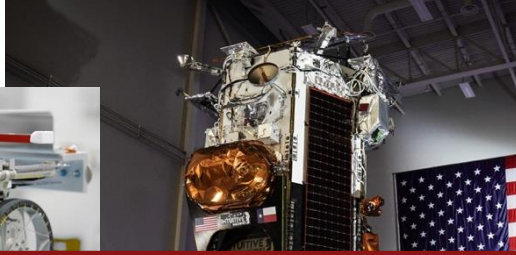


KAILA M. Y. COIMBRA AND GRACE GAO



Exploratory rover missions will serve as precursors to sustained surface operations.

2026



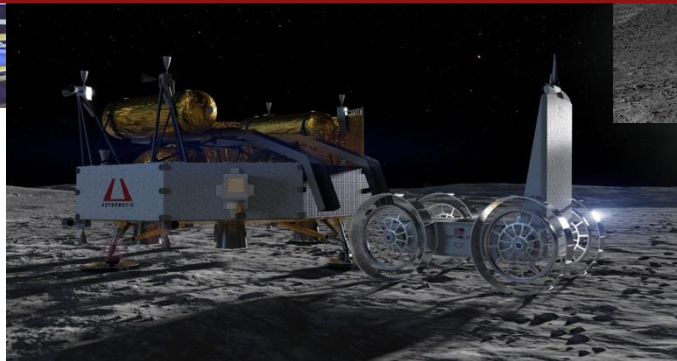
2027



Precise absolute localization of lunar rovers is critical to support long-range traverses, science targeting, and safe operations in challenging terrain.



*FLIP rover onboard
Griffin-1 lander^[2]*

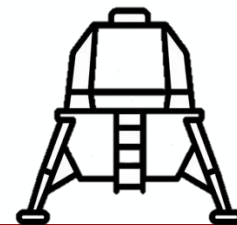


*VIPER rover onboard
Mark-1 lander^[3]*

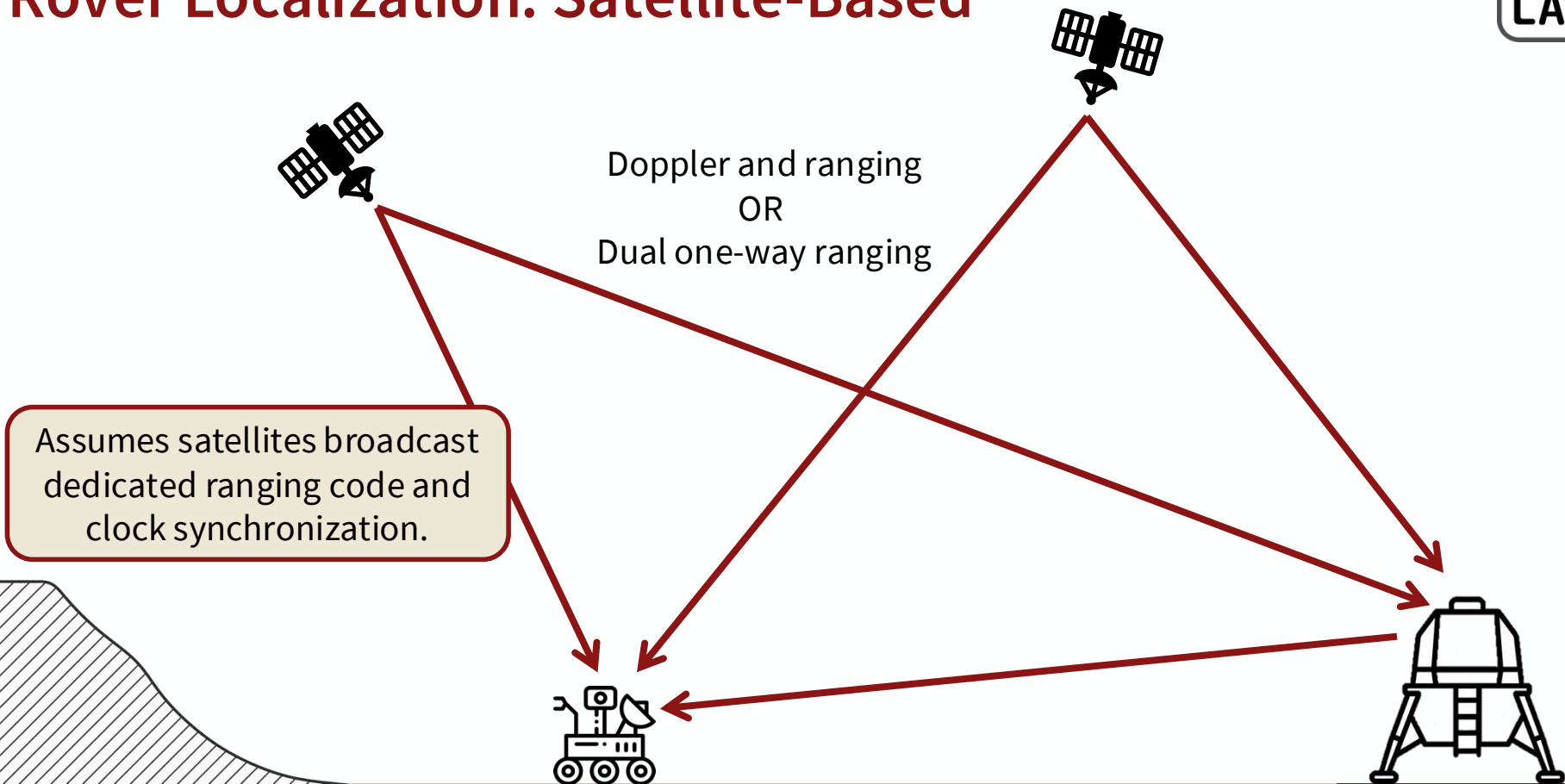
Rover Localization: Vision



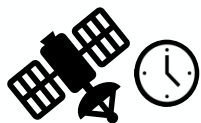
Depends on illumination
and map quality



Rover Localization: Satellite-Based

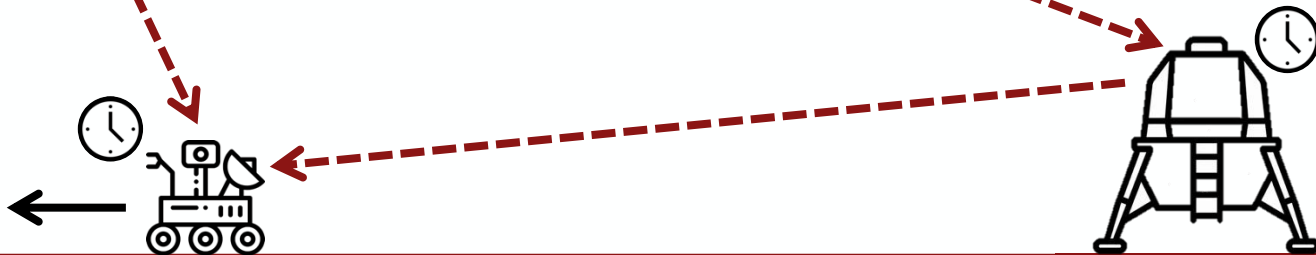


Rover Localization: Minimal Infrastructure

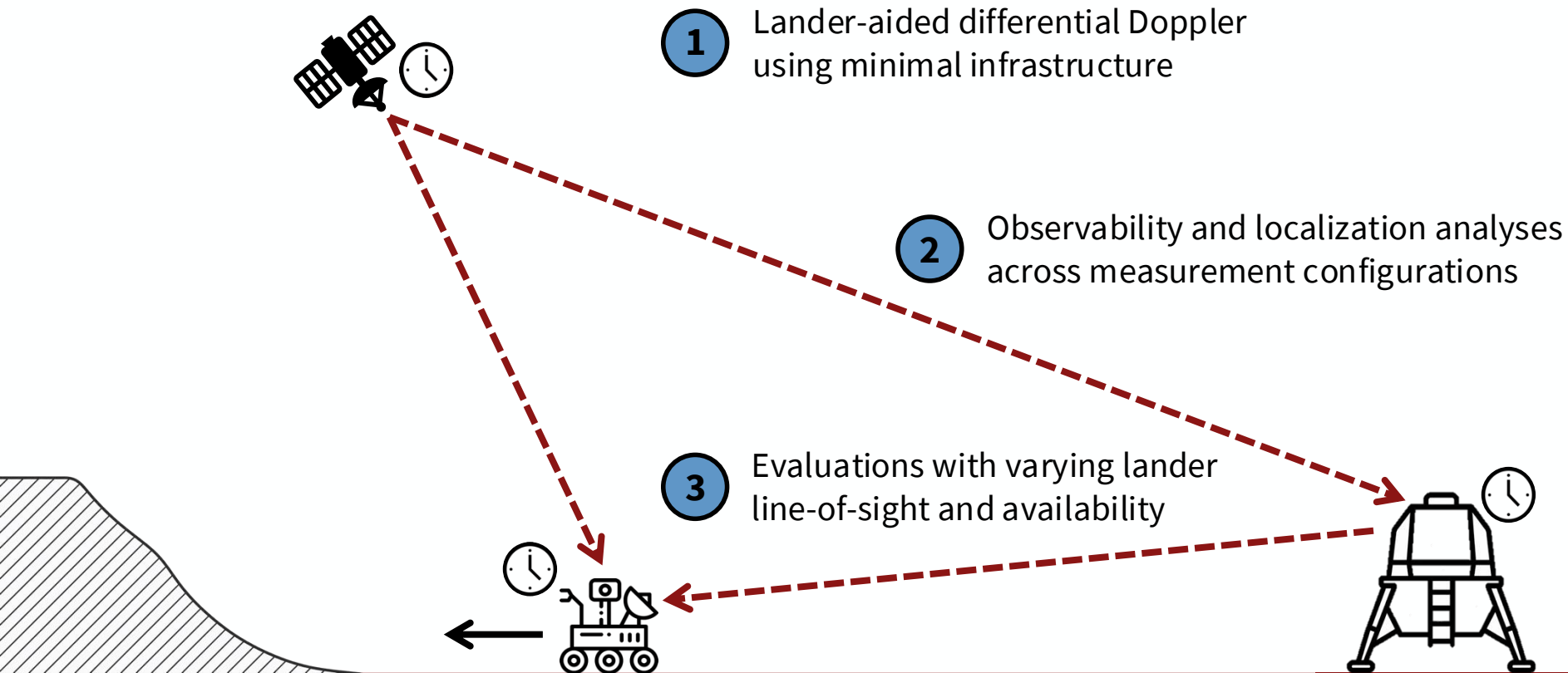


Constraints applied to this work

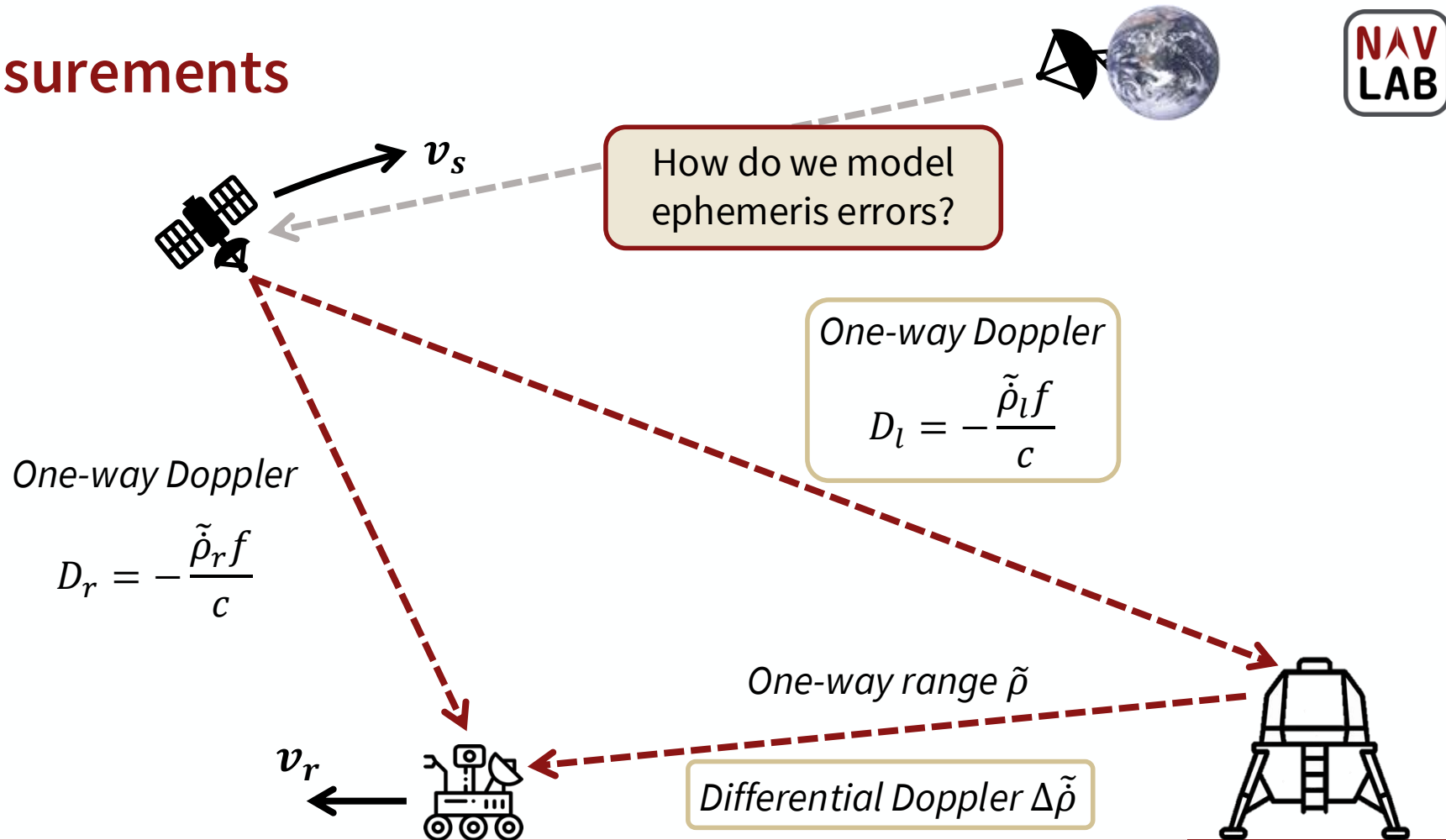
- Agnostic to illumination or map quality
- Only one satellite
- Satellite does not broadcast dedicated ranging code
- Consider clock bias/drift
- Rover is moving



Contributions



Measurements



Broadcast Ephemeris

Broadcast satellite state is imperfect.

$$\tilde{\mathbf{r}}_s = \mathbf{r}_s + \delta\mathbf{r}_s$$

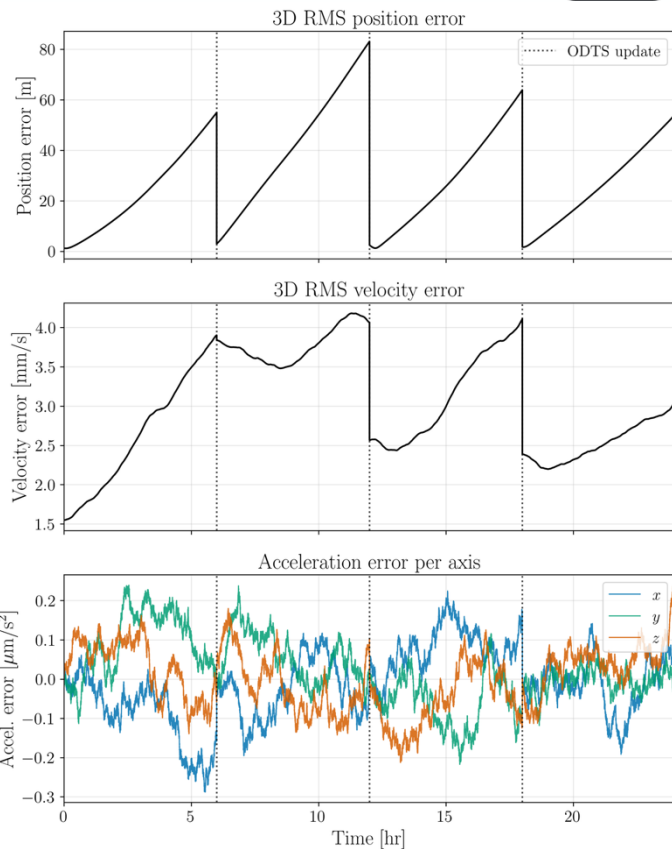
$$\tilde{\mathbf{v}}_s = \mathbf{v}_s + \delta\mathbf{v}_s$$

Apply a first-order Gauss-Markov (FOGM) process to acceleration error $\delta\mathbf{a}_s$ ^[9].

$$\delta\mathbf{x}_{eph} = \begin{bmatrix} \delta\mathbf{r}_s \\ \delta\mathbf{v}_s \\ \delta\mathbf{a}_s \end{bmatrix}$$

Source	State Estimated	Dimension
Satellite/Lander	$\mathbf{r}_r$	3
Satellite	$c(\dot{\delta}t_r - \dot{\delta}t_s)$	1
Lander	$c(\delta t_r - \delta t_l)$	1
	$c(\dot{\delta}t_r - \dot{\delta}t_l)$	1
Satellite	$\delta\mathbf{x}_{eph}$	9

How do we consider this state without estimating it?



Schmidt-Kalman Filter

Extended Kalman Filter^[10]

Predict

$$\hat{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1})$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1}$$

Update

$$\mathbf{y}_k = \mathbf{z}_k - h(\hat{\mathbf{x}}_{k|k-1})$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{y}_k$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{K}_k (\mathbf{H}_k \mathbf{P}_{k|k-1} + \mathbf{C}_k^T)$$

Schmidt^[11]: Consider effect of errors due to uncertain parameters.

Partition into estimated (x) and consider (c) blocks.

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_x \\ \mathbf{x}_c \end{bmatrix} \quad \mathbf{x}_c := \delta \mathbf{x}_{eph}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{xx} & \mathbf{P}_{xc} \\ \mathbf{P}_{cx} & \mathbf{P}_{cc} \end{bmatrix}$$

Force consider-block Kalman gain to zero.

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_x \\ \mathbf{K}_c \end{bmatrix} \quad \mathbf{K}_c = \mathbf{0}$$

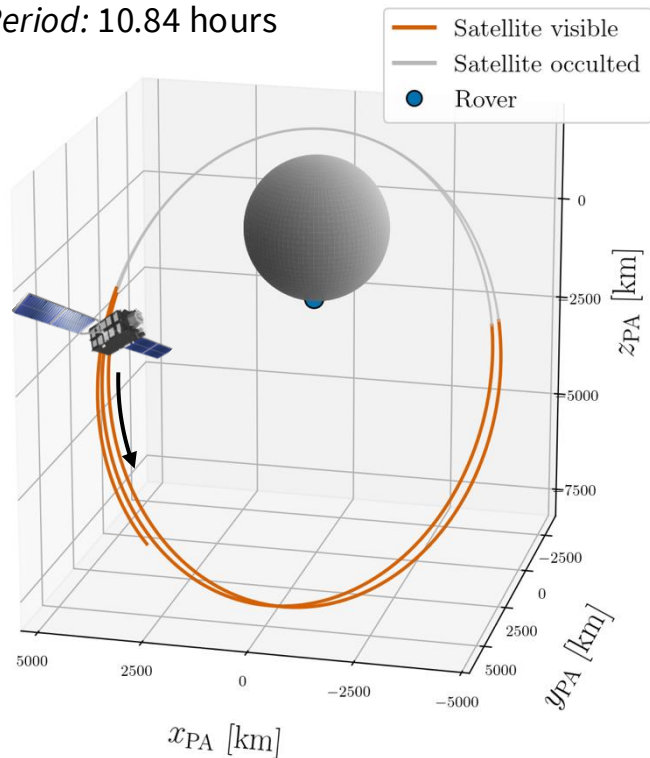
Considers the effect of ephemeris errors on the estimated states but does not correct the considered parameters.

Navigation Assets

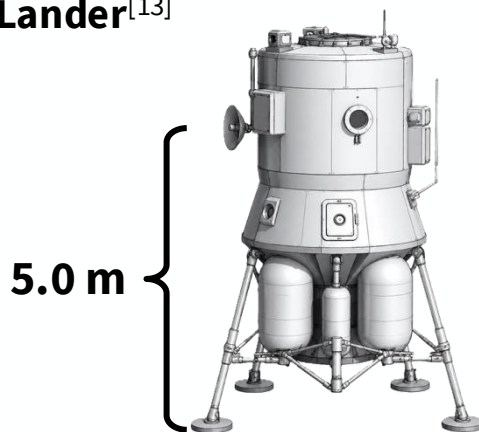
Relay Satellite^[12]

Elliptical Lunar Frozen Orbit (ELFO)

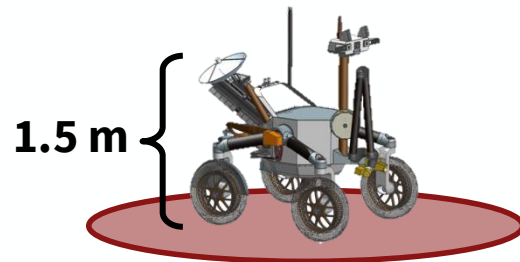
Period: 10.84 hours



Lander^[13]



Rover^[14]



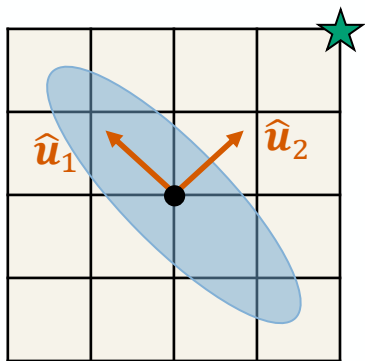
Artemis Basecamp

87.0° S, 50.5° E

Parameter		Value
Rover initial position σ per-axis	At lander	5 m
	Lost	100 m

Observability

2D Example



Define $\mathbf{G}_r$ as the effective Fisher information matrix for rover position.

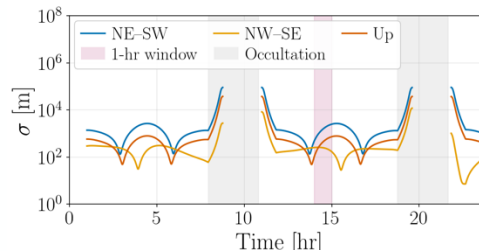
Observability metric:

$$\sigma^2(\hat{\mathbf{u}}) = \hat{\mathbf{u}}^T \mathbf{G}_r^{-1} \hat{\mathbf{u}}$$

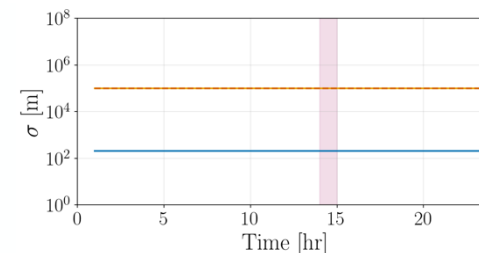
$$\sigma(\hat{\mathbf{u}}_1) > \sigma(\hat{\mathbf{u}}_2)$$

σ vs. time

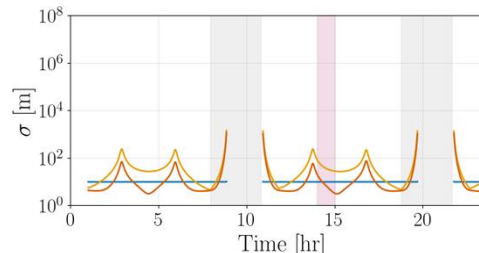
Satellite



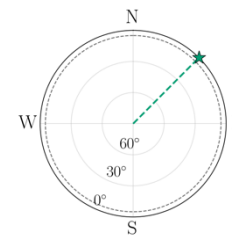
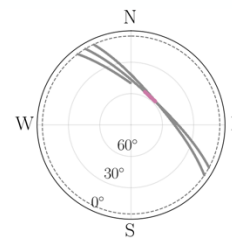
Lander



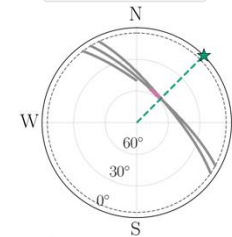
Satellite +
Lander



Sky plot



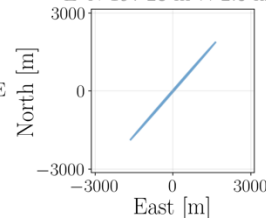
★ Lander direction



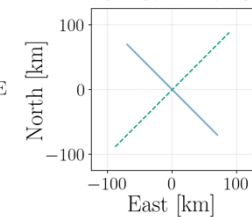
★ Lander direction

1σ ellipse at
1-hr window

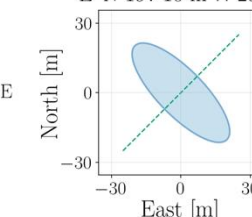
E-N 1σ : 28 m $\times$ 2.5 km



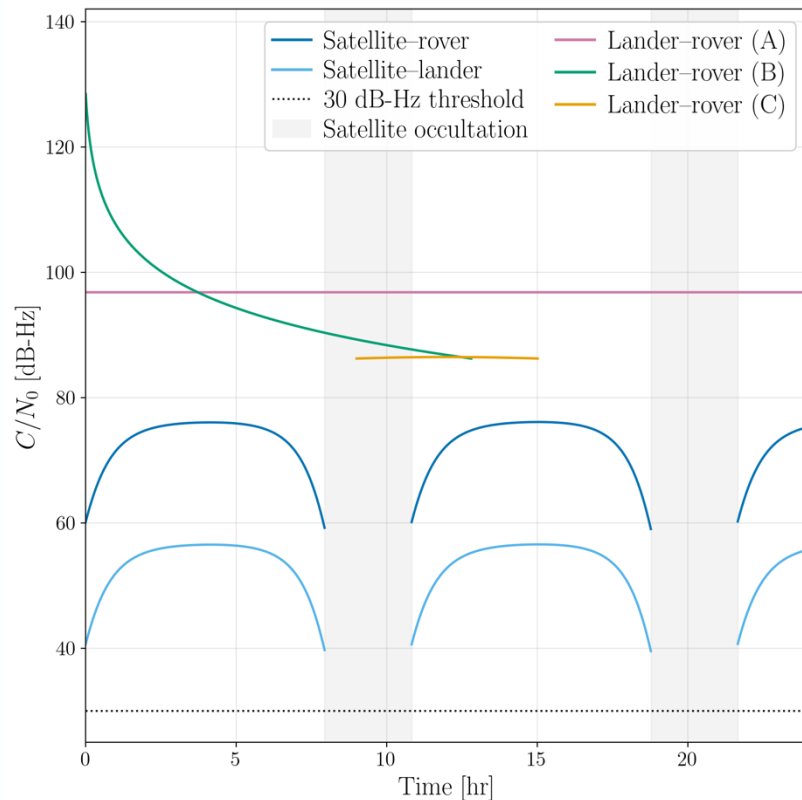
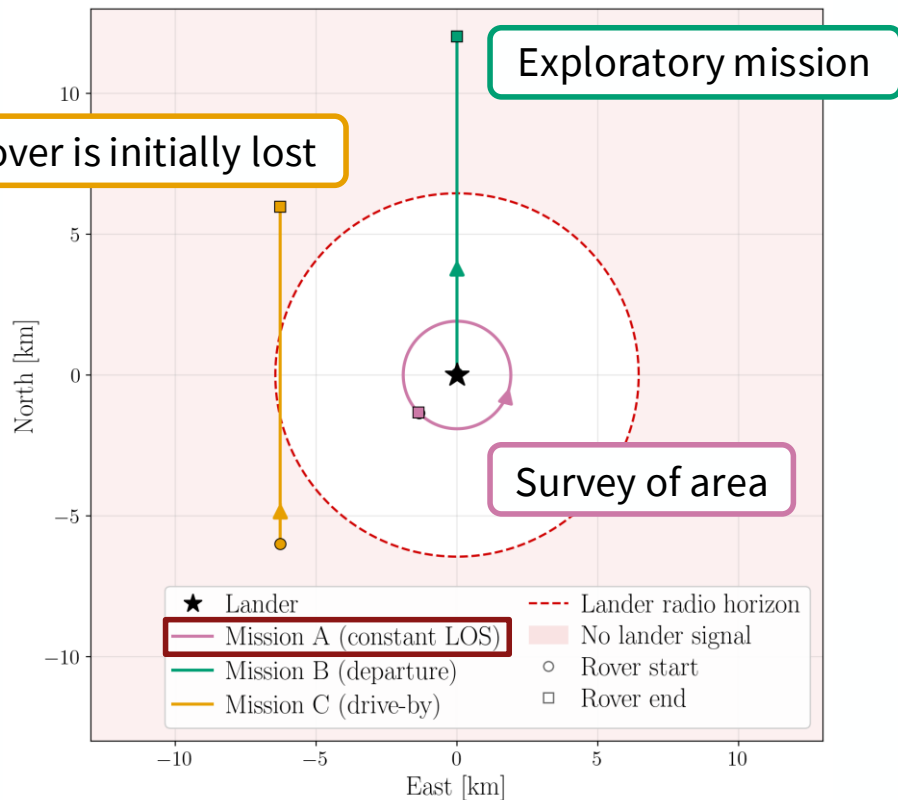
E-N 1σ : 207 m $\times$ >100 km



E-N 1σ : 10 m $\times$ 28 m



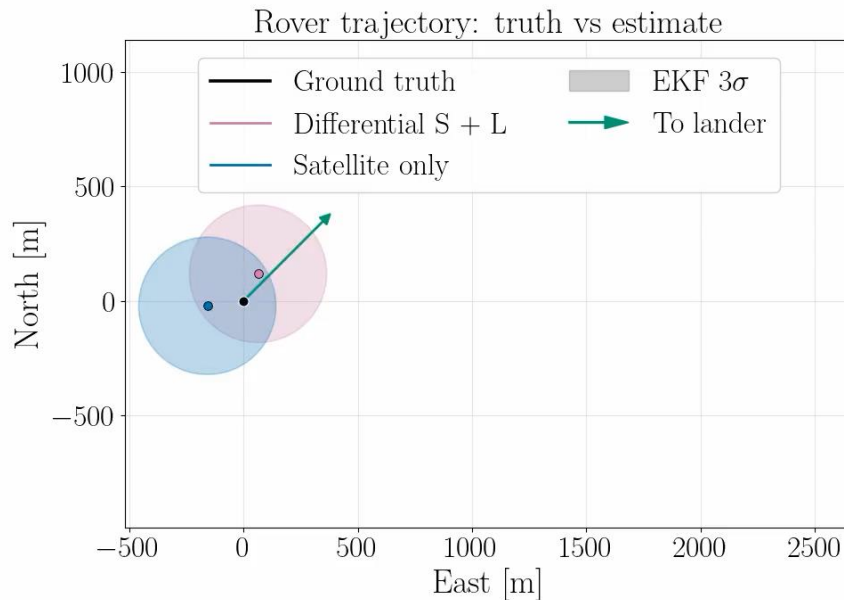
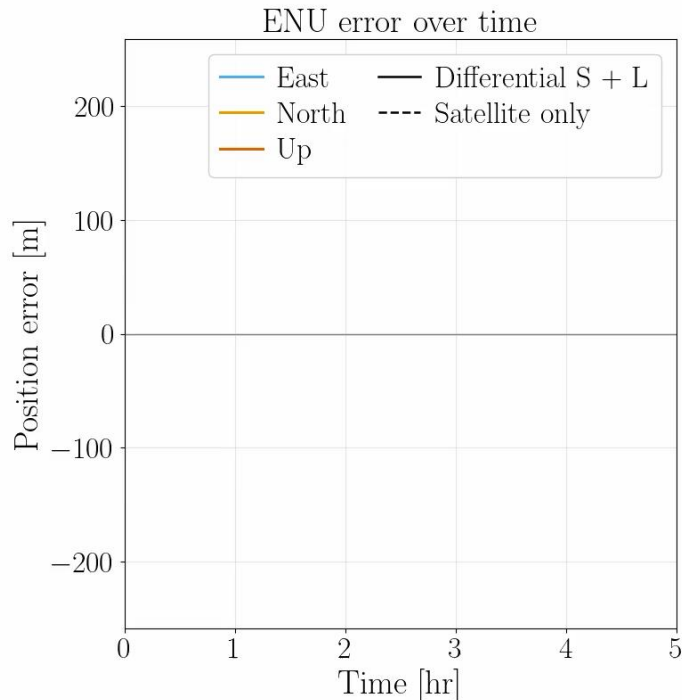
Mission Scenarios



Localization Performance with Diff S+L and S-only

Mission A: Constant LOS

Time: 0.00 h

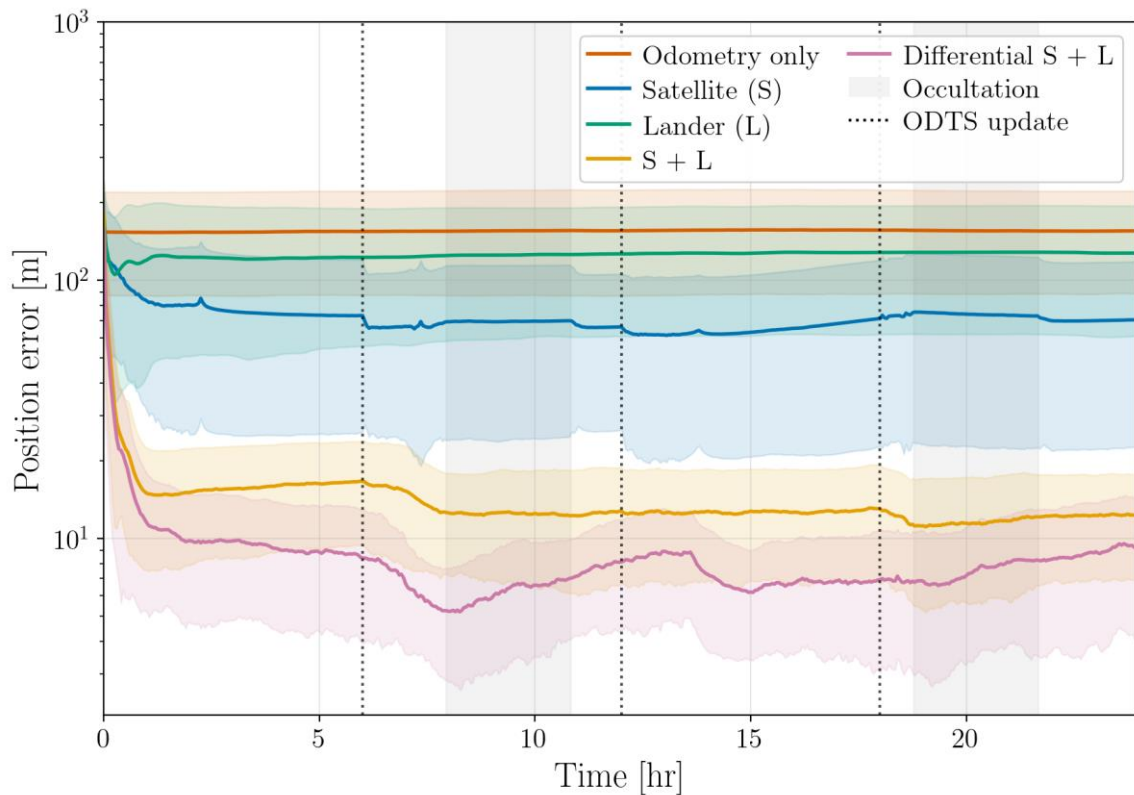


Ablation on Measurement Configuration

Mission A: Constant LOS

- Mean (solid line) and 1σ (shaded band) across 100 Monte Carlo runs

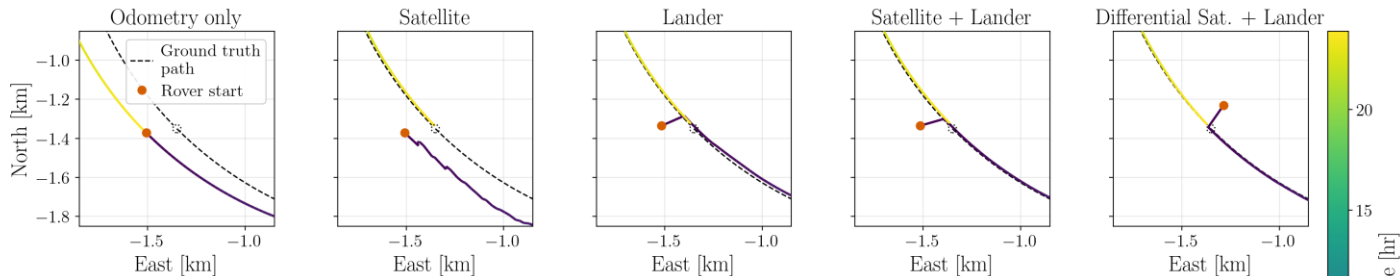
- Satellite-only outperforms lander-only case
- Differential S + L outperforms the rest, maintaining <10 m accuracy after 1.8 hours



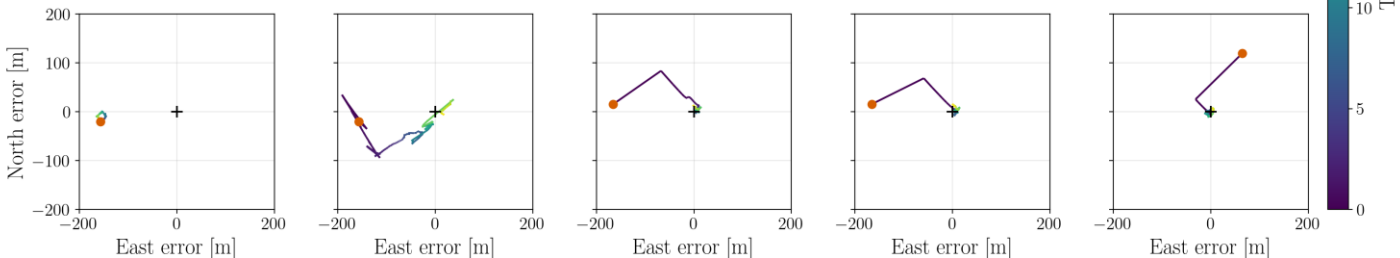
Ablation on Measurement Configuration (1 MC Run)

Measurement Configurations

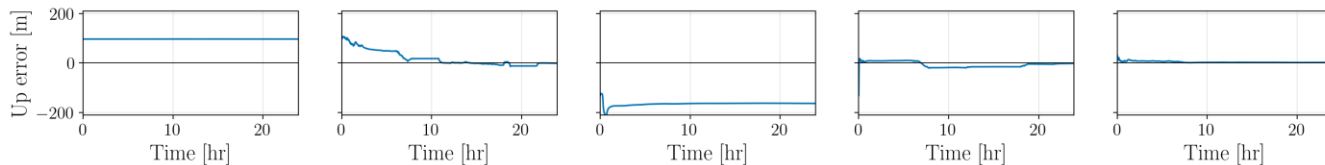
**Top-down
(zoomed)**



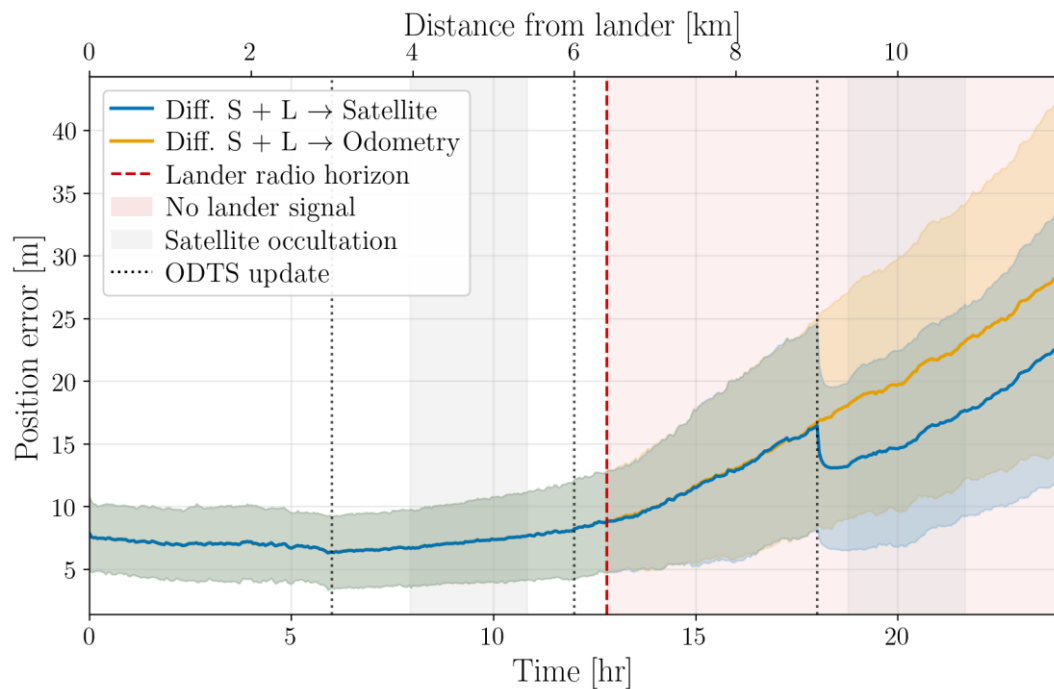
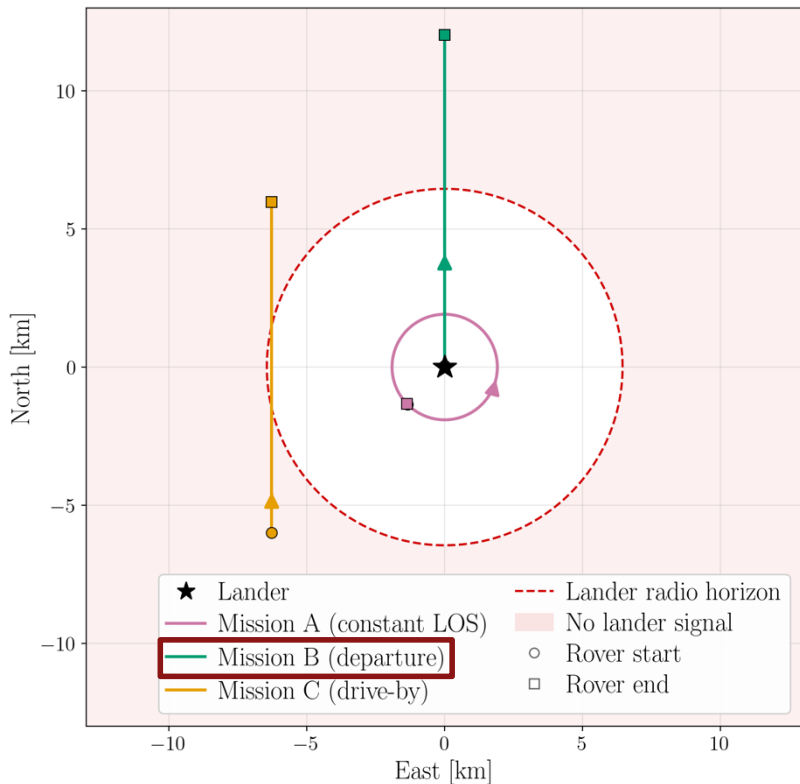
**Top-down
error**



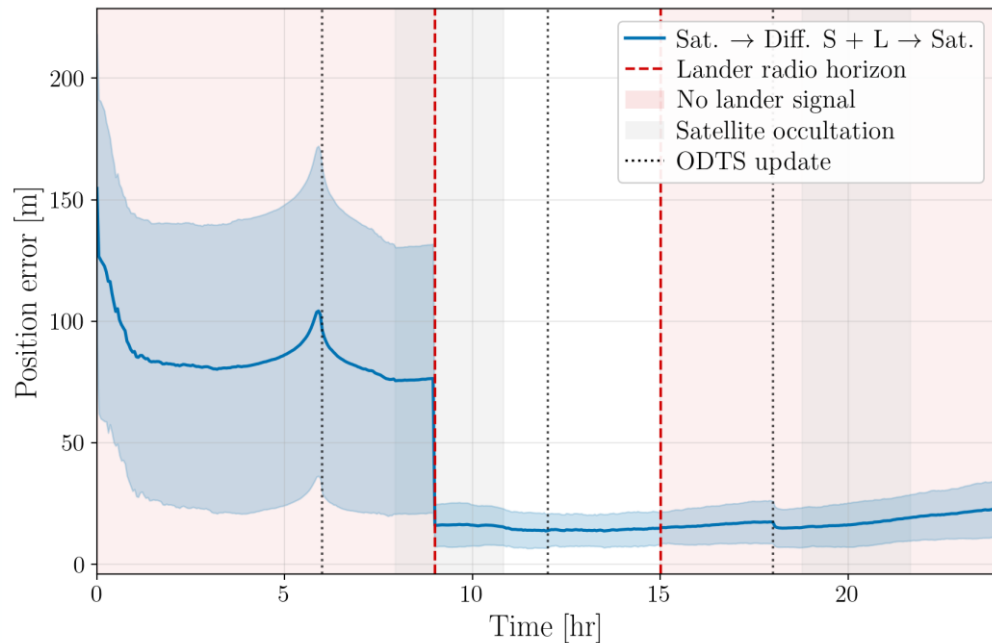
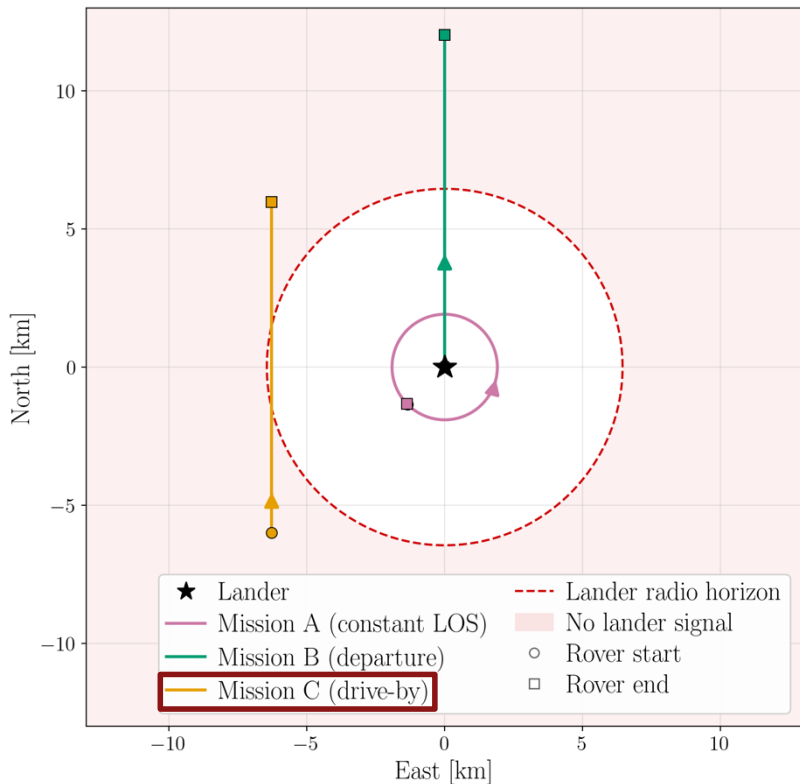
Up error



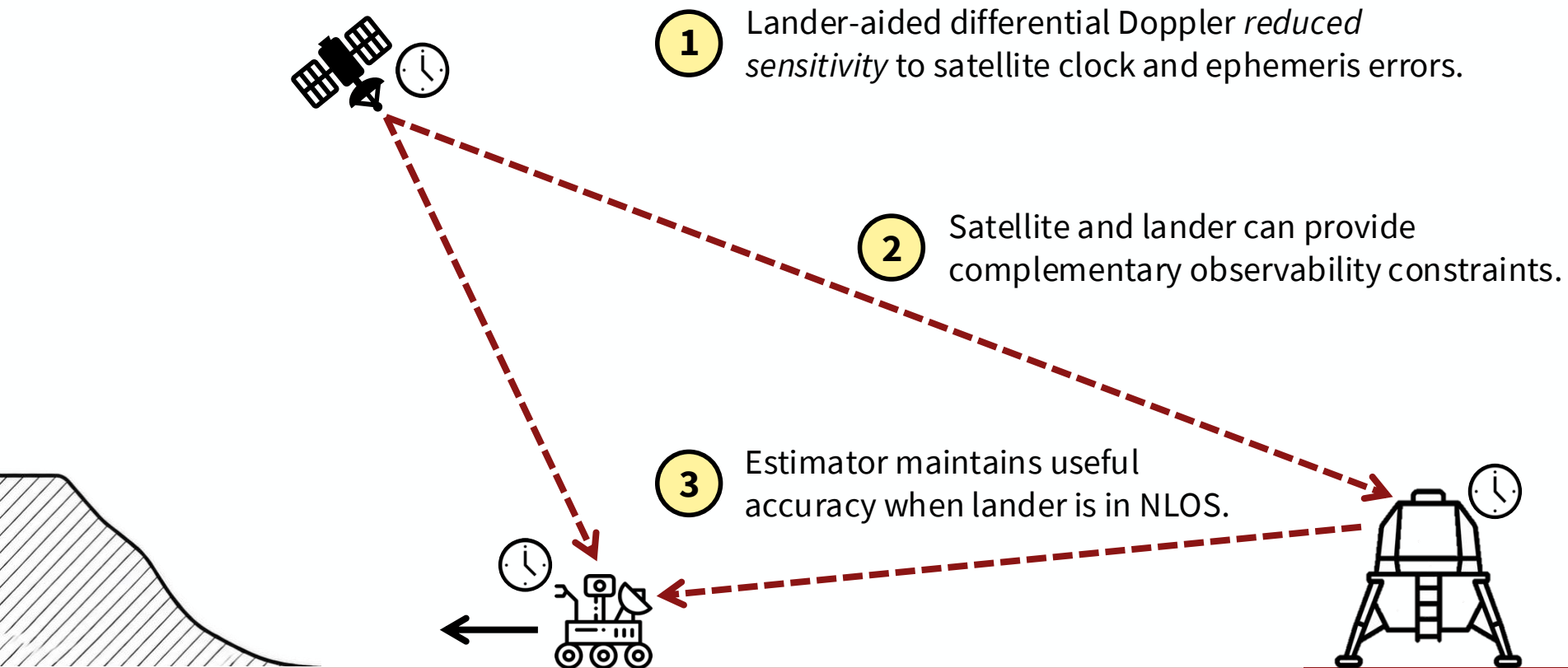
Localization Performance for Mission B



Localization Performance for Mission C



Conclusions



Acknowledgements



Knight-Hennessy Scholars
Stanford University



Graduate Research
Fellowship Program

BLUE ORIGIN



ION GNSS+ 2026 Papers from NAV Lab

- A1a: Mira Partha et al., “Bird’s-Eye-View Mapping from Vehicle Cameras via 3D Foundation Models” @ **8:57 AM on Wednesday (Windsong 1-2)**
- E1: Kaila Coimbra et al., “Time-Correlated Ephemeris Errors and Relativistic Effects for Single-Satellite Doppler Localization of Lunar Rovers” @ **10:40 AM on Wednesday (Windsong 7-8)**
- F2: Asta Wu et al., “Integrity Risk Mapping for Signals-of-Opportunity-based Urban UAS Navigation” @ **2:12 PM on Wednesday (Windsong 1-2)**
- D2b: Matthias Kura et al., “Optimal Reconfiguration of Lunar Navigation Constellations” @ **5:08 PM on Wednesday (Windsong 5-6)**
- E4: Matthias Kura et al., “Sidelobe vs. Side of Main Lobe: LuGRE Experiment Data Analysis and Filtering” @ **4:23 PM on Thursday (Windsong 11-12)**

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Thank you!