

Time-Correlated Ephemeris Errors and Relativistic Effects for Single-Satellite Doppler Localization of Lunar Rovers



KAILA M. Y. COIMBRA AND GRACE GAO



Need for Absolute Localization of Rovers

CADRE, 2026^[1]



FLEX, 2026^[2]



LUPEX, 2028^[3]



Endurance (concept), 2030^[4]



Precise absolute localization of lunar rovers in real time is essential.



A fully operational navigation constellation system will not be available for early-stage missions.

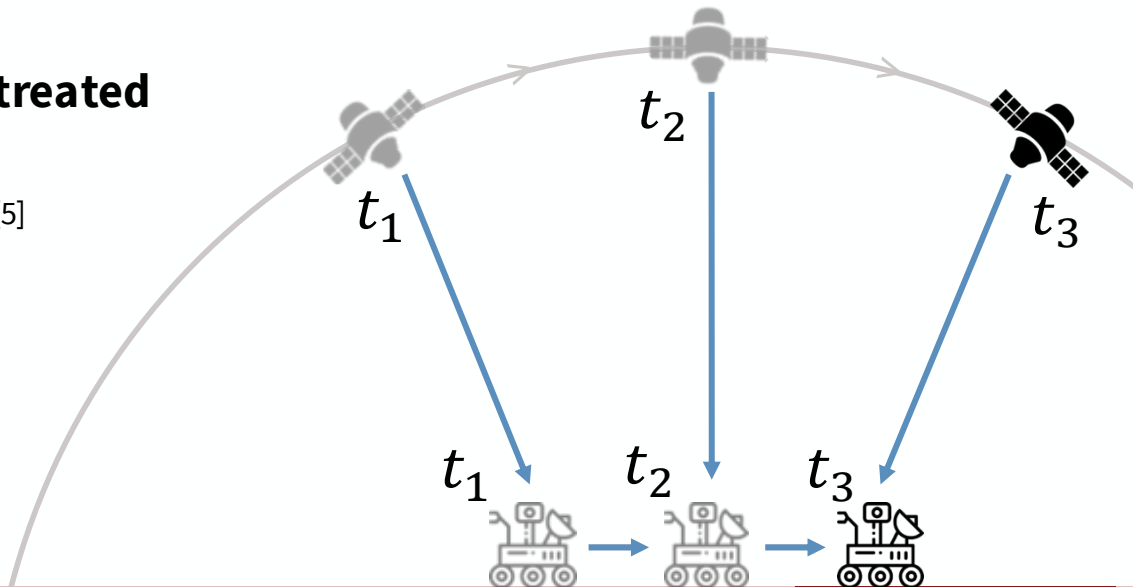
Rover Localization using Minimal Infrastructure

- Leverage Doppler shift from the communication signal of a single satellite
- Use the rover's onboard **odometry**, **Doppler shift**, and the satellite's **broadcast ephemeris** to localize the rover

Error sources that are often under-treated in lunar surface navigation studies:

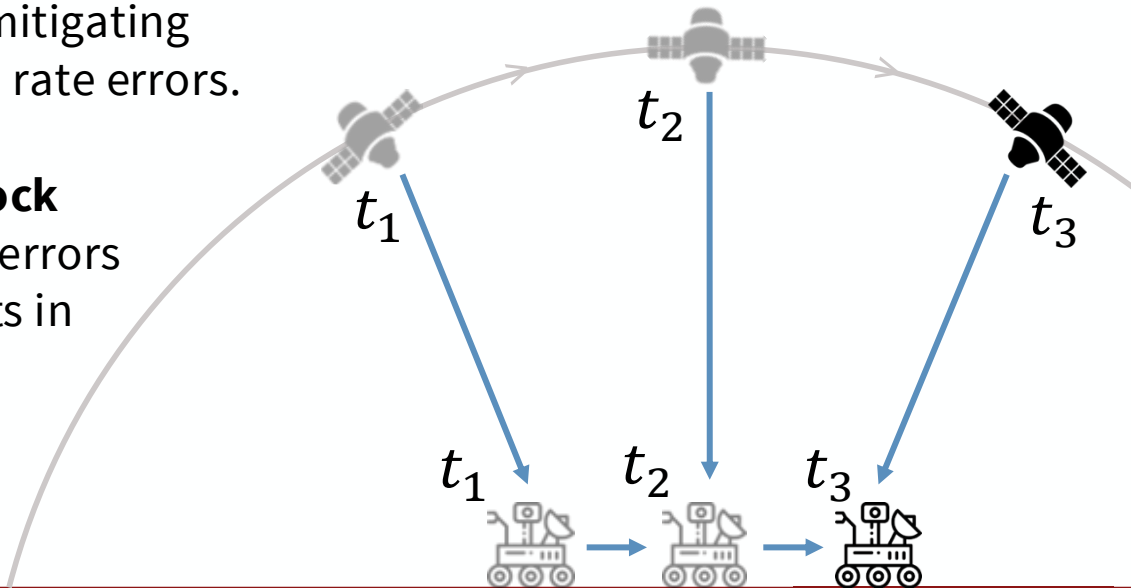
- Time-correlated ephemeris errors^[5]
- Relativistic clock rate effects^[6, 7]

Can accumulate 10 to 100 m positioning errors in hours



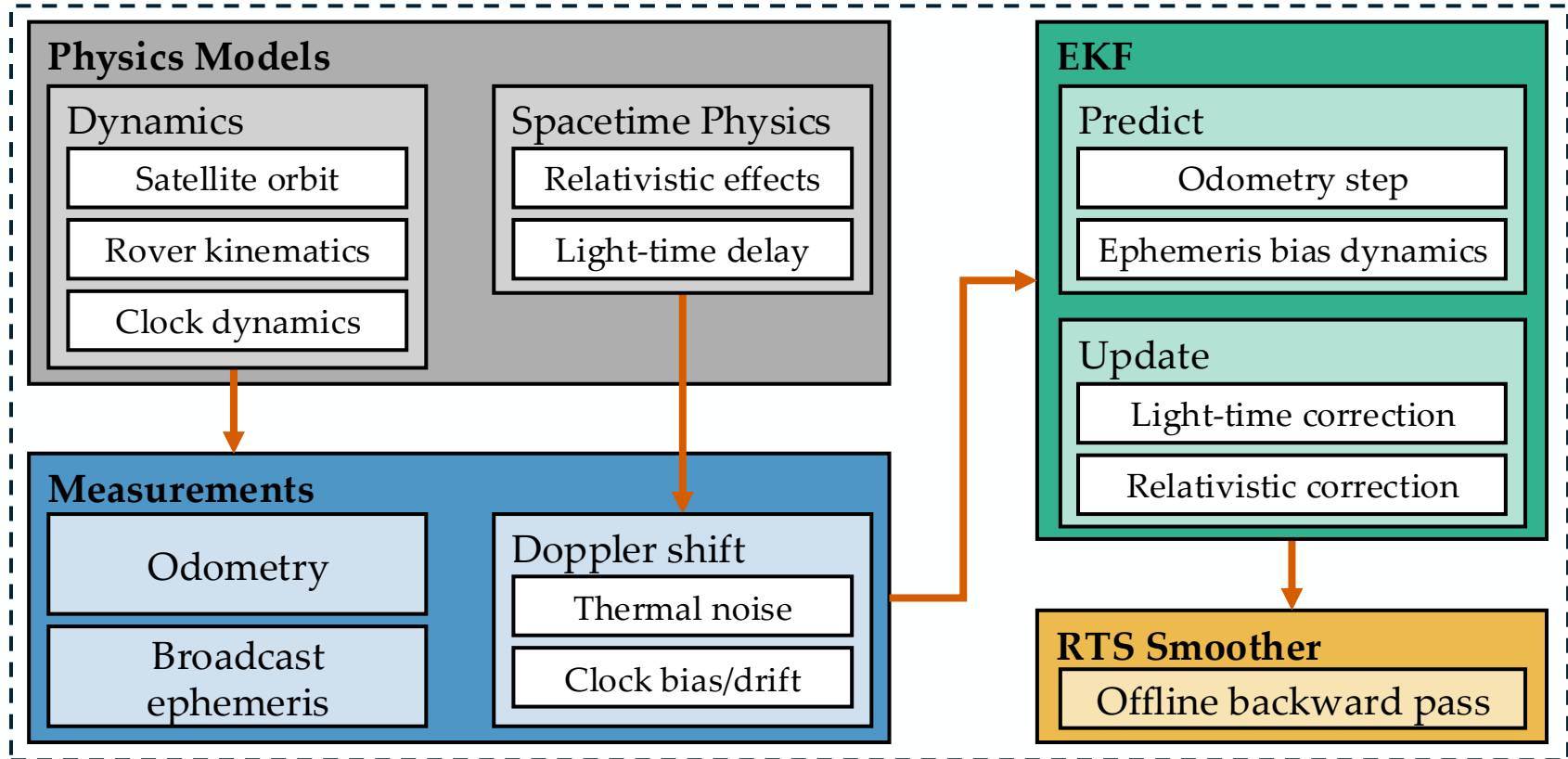
Contributions

- **Characterization of time-correlated ephemeris errors** in single-satellite Doppler-aided lunar rover localization.
- **A reduced-order EKF and Rauch-Tung-Striebel (RTS) smoothing approach** for mitigating ephemeris-induced pseudorange rate errors.
- **Quantification of relativistic clock rate effects** and the localization errors caused by neglecting these effects in the filter.



Approach

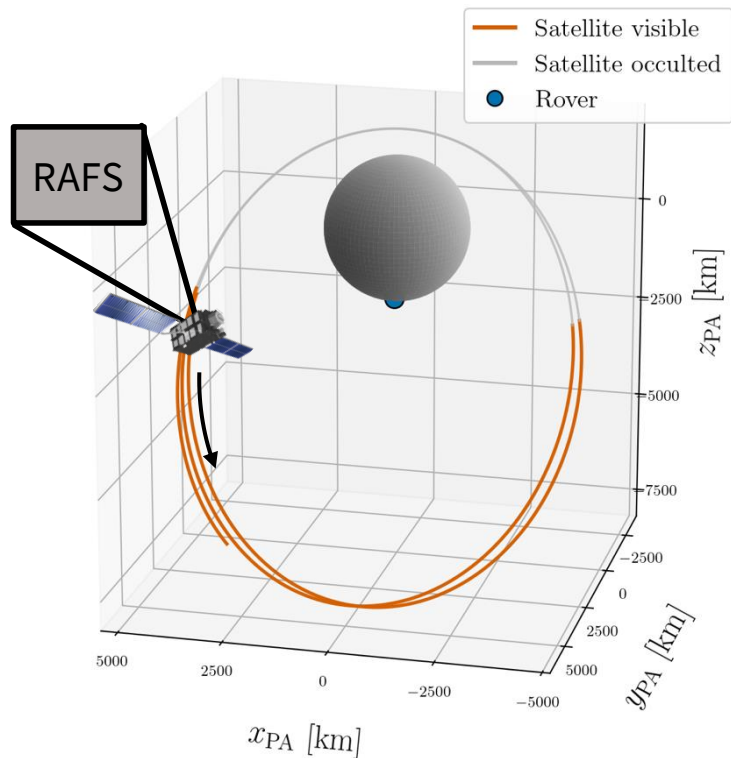
N Monte Carlo realizations



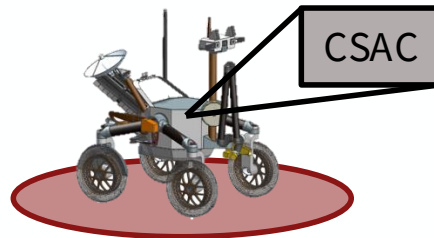
Dynamics

Elliptical Lunar Frozen Orbit (ELFO)^[8]

Orbital period: 10.84 hours



Sample Return Rover^[9]



Artemis Basecamp

87.0° S, 50.5° E

Parameter		Value
Initial σ per-axis	Position	5 m
	Velocity	1 mm/s
Rover speed		0.5 km/hr

Rover Onboard Odometry

$$\Delta d_{meas} = \Delta d_{true} + \varepsilon_{odo} \quad , \quad \varepsilon_{odo} \sim \mathcal{N}(0, \sigma_{odo}^2)$$

$$\sigma_{odo} = \alpha_{odo} \cdot \Delta d_{true}$$

1

Observed Doppler Shift

Doppler shift

$$D = -\frac{\dot{\rho}f}{c} \quad \textcircled{2}$$

Light-time corrected states $\mathbf{x}_s^* := \mathbf{x}_s(t_{tx})$

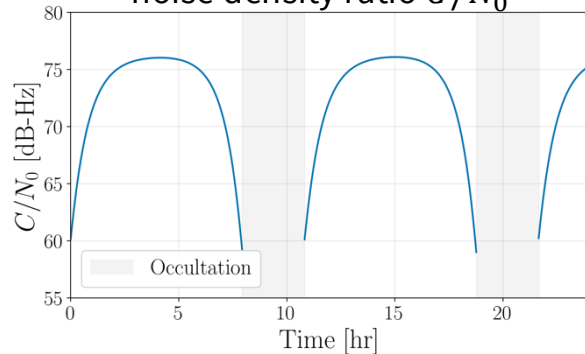
Observed pseudorange rate

$$\tilde{\rho}(t) = \underbrace{(\mathbf{v}_s^* - \mathbf{v}_r) \cdot \frac{\mathbf{r}_s^* - \mathbf{r}_r}{\|\mathbf{r}_s^* - \mathbf{r}_r\|}}_{\text{True range rate}} + \underbrace{c(\dot{\delta}t_r - \dot{\delta}t_s)}_{\text{Relative drift}} + \underbrace{\varepsilon_{\dot{\rho}}}_{\text{Thermal noise}}, \varepsilon_{\dot{\rho}} \sim \mathcal{N}(0, \sigma_t^2)$$

Stochastic

$$\dot{\delta}t = \dot{\delta}t_{stoch} + \underbrace{\dot{\delta}t_{rel}}_{\text{Deterministic}}$$

σ_t is a function of carrier-to-noise density ratio C/N_0



Relativistic Effects

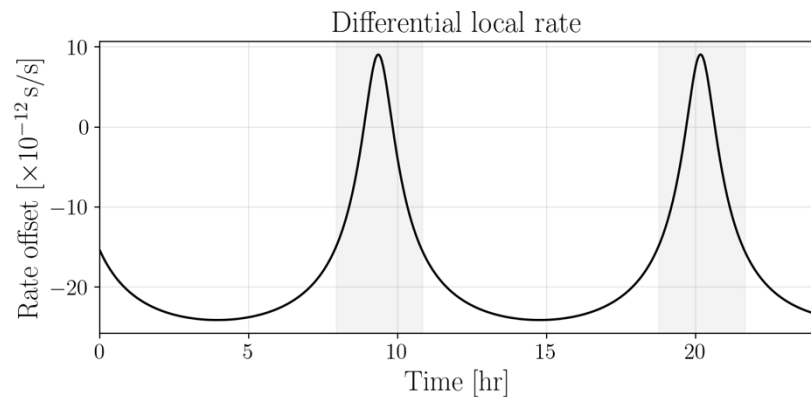
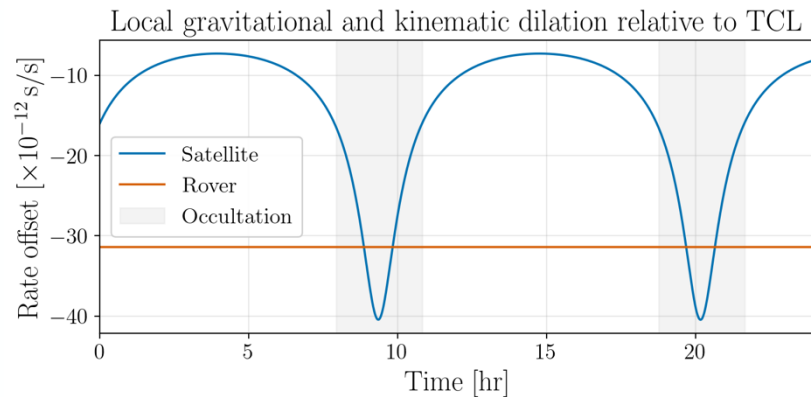
Proper time is time measured by an idealized clock along its worldline

Must convert to a common reference, *Lunar Coordinate Time (TCL)*

$$\dot{\delta t}_{rel} = \frac{d(\overbrace{\tau_L}^{\text{Proper time}} - \overbrace{s}^{\text{TCL time}})}{ds} \approx -\frac{1}{c^2} \left(\underbrace{\frac{\dot{r}_L^2}{2}}_{\text{Kinematic}} - \underbrace{\frac{\mu_L}{r_L}}_{\text{Gravitational}} \right)$$

Differential local rate

$$\eta_{rel} = \dot{\delta t}_{rel}(\mathbf{r}_r, \mathbf{v}_r) - \dot{\delta t}_{rel}(\mathbf{r}_s, \mathbf{v}_s)$$



Broadcast Ephemeris

Broadcast satellite state is imperfect.

$$\begin{aligned}\tilde{\mathbf{r}}_s &= \mathbf{r}_s + \delta\mathbf{r}_s \\ \tilde{\mathbf{v}}_s &= \mathbf{v}_s + \delta\mathbf{v}_s\end{aligned}$$

3

Apply a first-order Gauss-Markov (FOGM) process to acceleration error $\delta\mathbf{a}_s$.

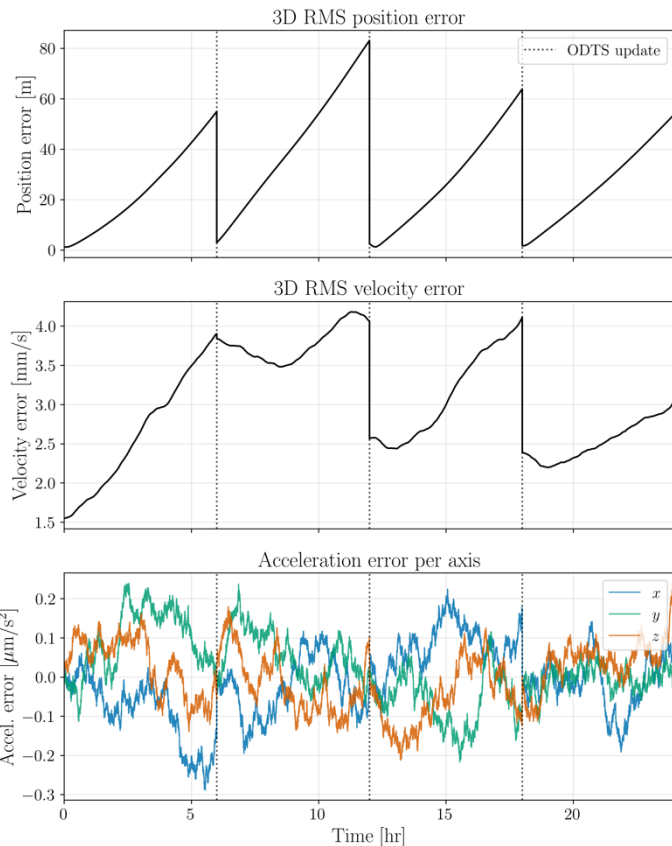
$$\mathbf{x}_{eph} = \begin{bmatrix} \delta r_{s,i} \\ \delta v_{s,i} \\ \delta a_{s,i} \end{bmatrix}$$

$i \in \{x, y, z\}$

$$\mathbf{x}_{eph,k+1} = \Phi_{eph}(\Delta t_k) \mathbf{x}_{eph,k} + \mathbf{w}_{eph,k}$$

$$\Phi_{eph}(\Delta t) = \begin{bmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & e^{-\Delta t/\tau_{eph}} \end{bmatrix}$$

Parameter	Value
Correlation time τ_{eph}	2 hours
Steady-state acceleration σ per-axis	10^{-4} mm/s ²
Reset time interval	6 hours



State Estimation

State: $\mathbf{x} = \begin{bmatrix} \mathbf{r}_r \\ d_{clk} \end{bmatrix} \in \mathbb{R}^4$
 $d_{clk} = c(\dot{\delta}t_r - \dot{\delta}t_s)$



Ephemeris errors enter the Doppler observable as a time-correlated measurement bias.

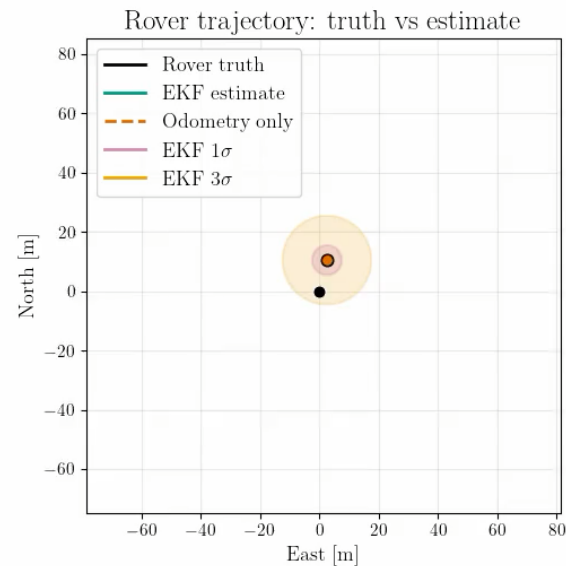
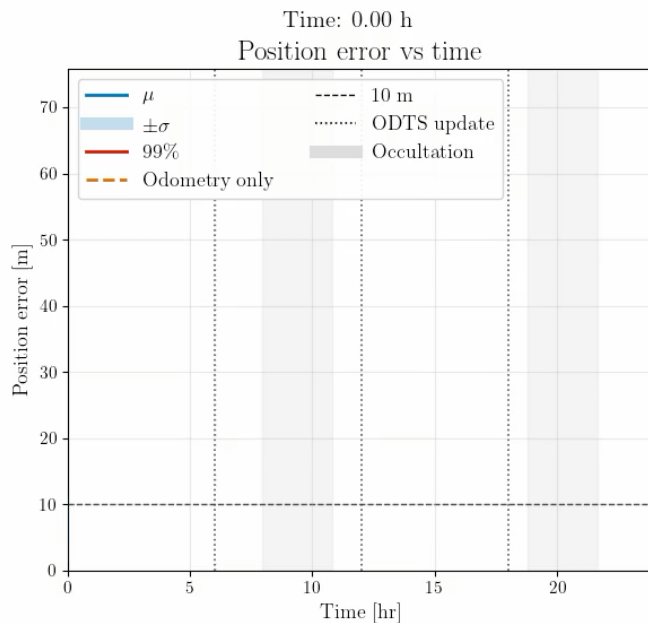
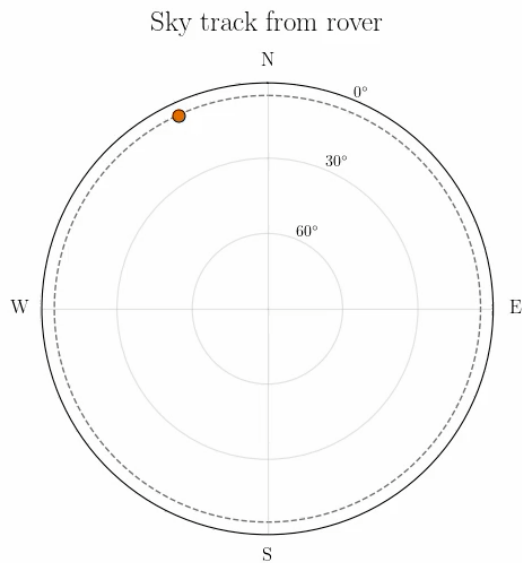
$$\mathbf{x} = \begin{bmatrix} \mathbf{r}_r \\ d_{clk} \\ b_{eph} \end{bmatrix} \in \mathbb{R}^5$$

Predict: $\hat{\mathbf{x}}_{k|k-1} = \begin{cases} \hat{\mathbf{r}}_{r,k-1} + \Delta d_{meas,k-1} \mathbf{u}_{hdg,k-1} \\ \hat{d}_{clk,k-1|k-1} \\ e^{-\Delta t / \tau_{eph}} \hat{b}_{eph,k-1|k-1} \end{cases}$ Model bias using FOGM dynamics^[11].

Update: $\hat{\rho}_k = (\tilde{\mathbf{v}}_s^* - \hat{\mathbf{v}}_{r,k}) \cdot \frac{\tilde{\mathbf{r}}_s^* - \hat{\mathbf{r}}_{r,k}}{\|\tilde{\mathbf{r}}_s^* - \hat{\mathbf{r}}_{r,k}\|} + \hat{d}_{clk,k|k-1} + \hat{b}_{eph,k|k-1}$ Predicted pseudorange

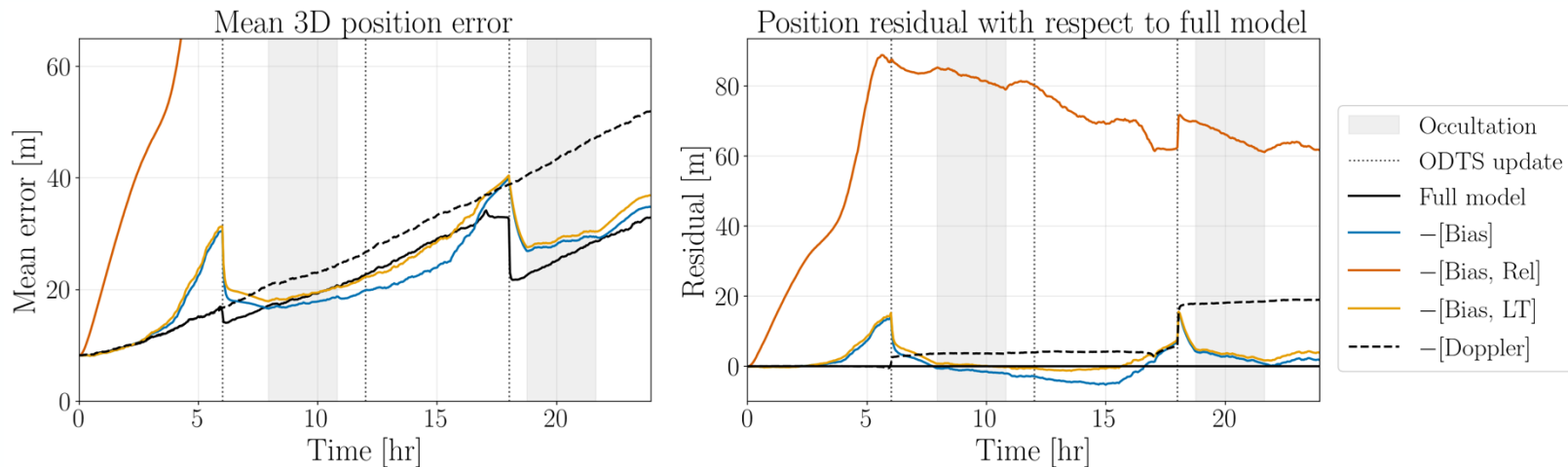
$\mathbf{y}_k = \tilde{\rho} - (\hat{\rho}_k + c\eta_{rel,k})$ Innovation

Forward EKF Pass with Full Filter Design



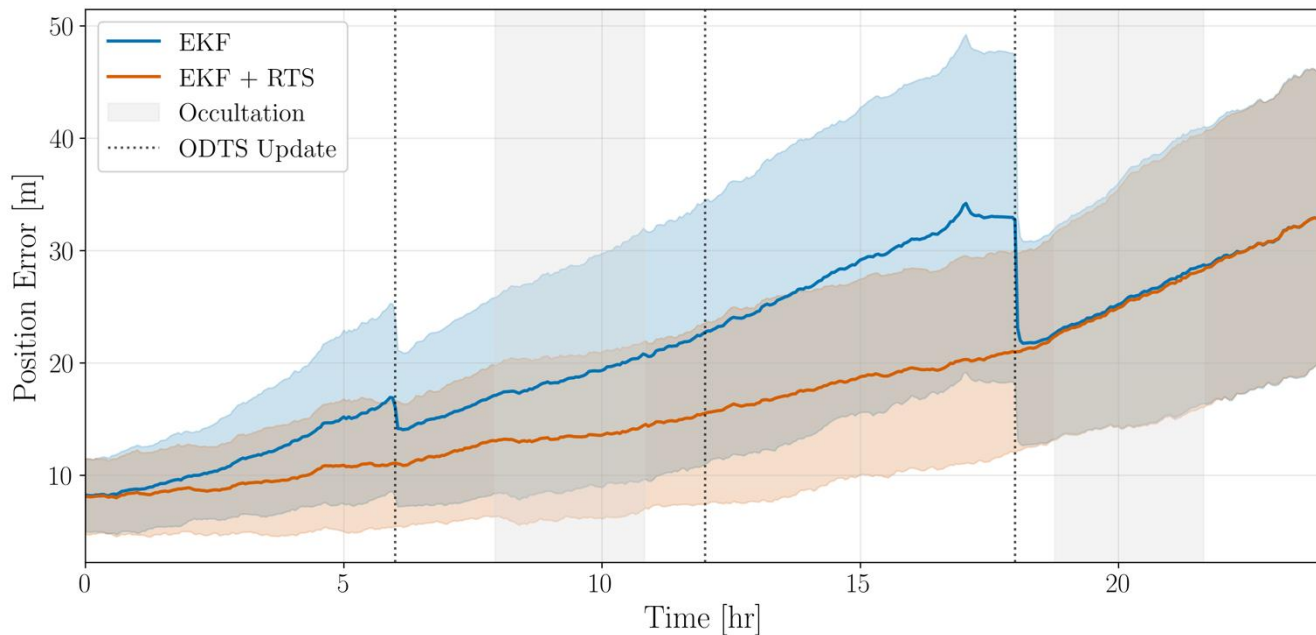
Doppler + odometry weakly constrain the rover heading direction, causing error growth between ODS updates. Improvement over odometry occurs mainly at each update.

Ablation Study on Filter Design



Ablation Case	Final mean position error [m]	Minimum residual [m]	Maximum residual [m]
Full model	32.9	---	---
-[Bias]	34.8	-5.2	15.2
-[Bias, Rel]	94.7	0.0	88.9
-[Bias, LT]	36.9	-1.3	15.6
-[Doppler]	52.0	-0.5	19.1

Forward EKF Pass with RTS Smoothing



The backward pass is able to mitigate the errors due to ephemeris errors and take better advantage of the ODS resets for all previous state estimates.

Acknowledgements



Knight-Hennessy Scholars
Stanford University



Graduate Research
Fellowship Program

BLUE ORIGIN



ION GNSS+ 2026 Papers from NAV Lab

- A1a: Mira Partha et al., “Bird’s-Eye-View Mapping from Vehicle Cameras via 3D Foundation Models” @ **8:57 AM on Wednesday (Windsong 1-2)**
- F2: Asta Wu et al., “Integrity Risk Mapping for Signals-of-Opportunity-based Urban UAS Navigation” @ **2:12 PM on Wednesday (Windsong 1-2)**
- G2a: Kaila Coimbra et al., “Lander-Aided Differential Doppler and One-Way Ranging for Lunar Rover Navigation with a Single Relay Satellite” @ **2:58 PM on Wednesday (Palm)**
- D2b: Matthias Kura et al., “Optimal Reconfiguration of Lunar Navigation Constellations” @ **5:08 PM on Wednesday (Windsong 5-6)**
- E4: Matthias Kura et al., “Sidelobe vs. Side of Main Lobe: LuGRE Experiment Data Analysis and Filtering” @ **4:23 PM on Thursday (Windsong 11-12)**

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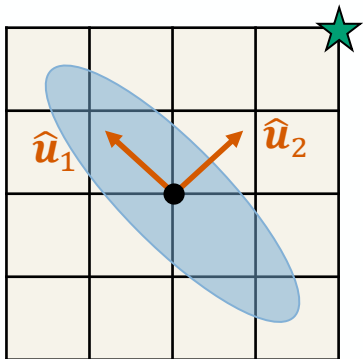
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Thank you!

Observability

2D Example

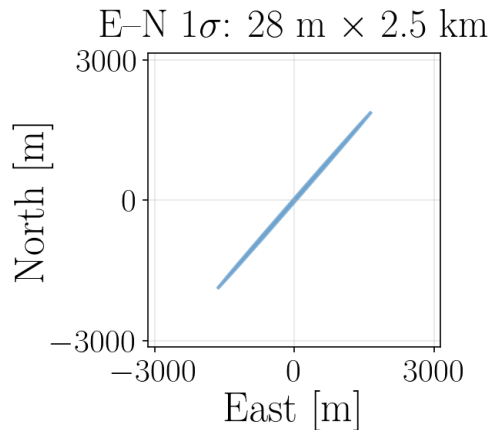
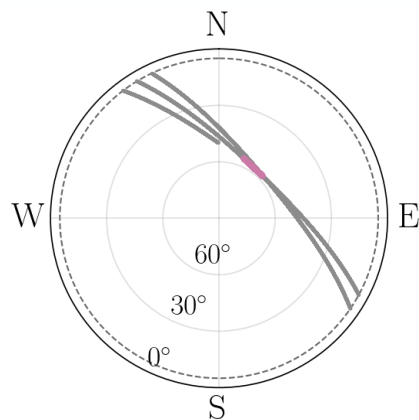
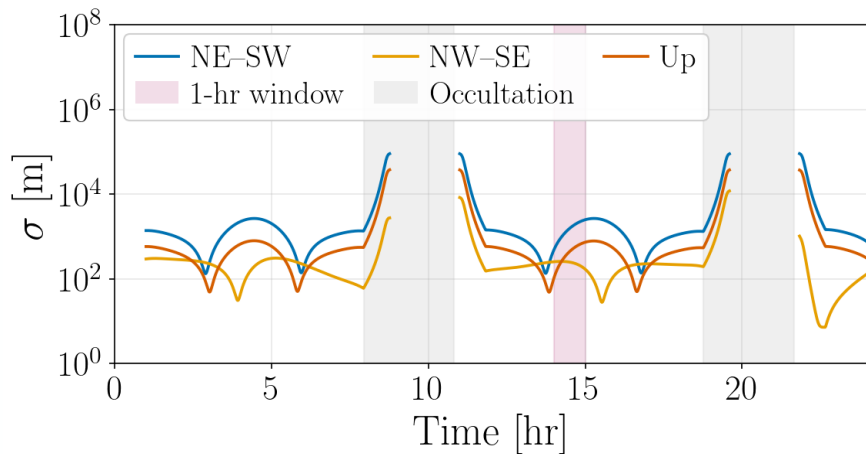


Define $\mathbf{G}_r$ as the effective Fisher information matrix for rover position.

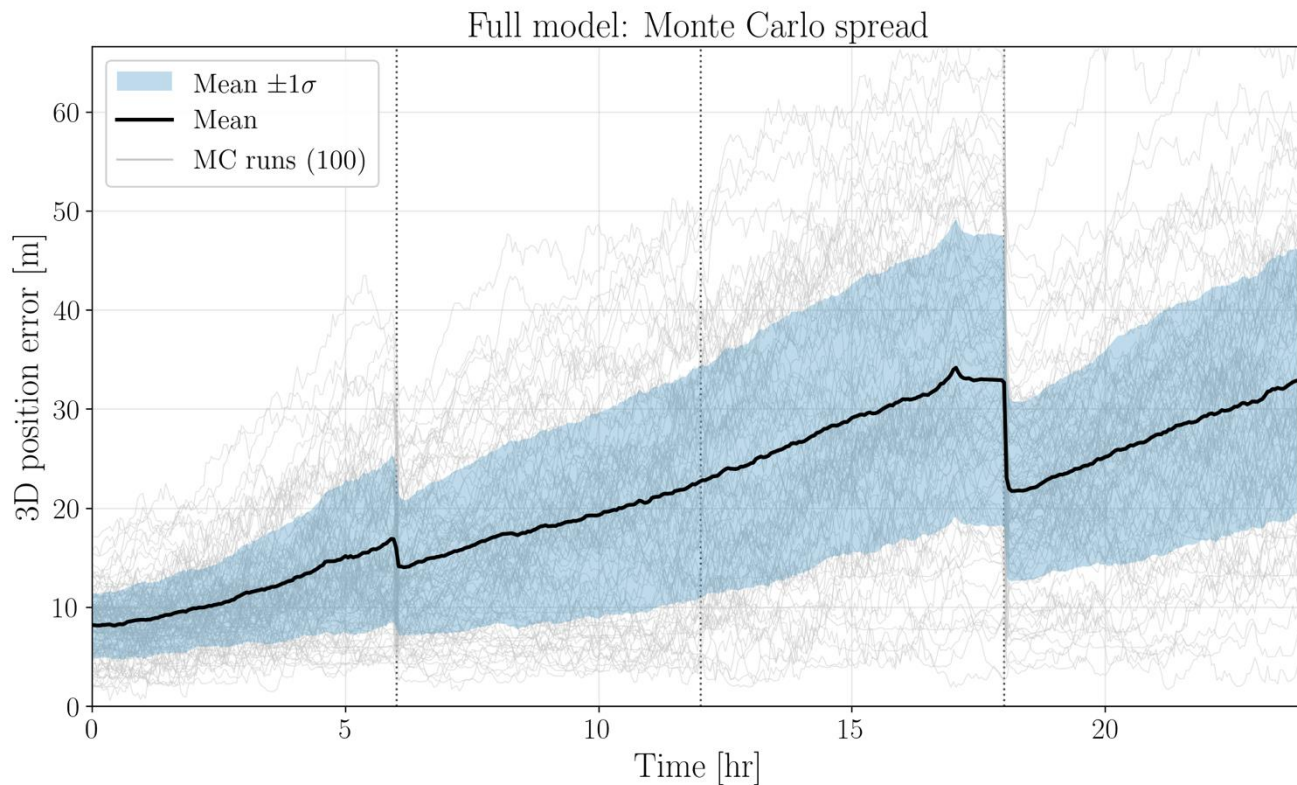
Observability metric:

$$\sigma^2(\hat{\mathbf{u}}) = \hat{\mathbf{u}}^T \mathbf{G}_r^{-1} \hat{\mathbf{u}}$$

$$\sigma(\hat{\mathbf{u}}_1) > \sigma(\hat{\mathbf{u}}_2)$$



Full Model (all MC runs)



ENU Decomposition

ENU position error, median MC run per case

