Comparative Analysis and Design of a Dual-Satellite System for Lunar Rover Localization

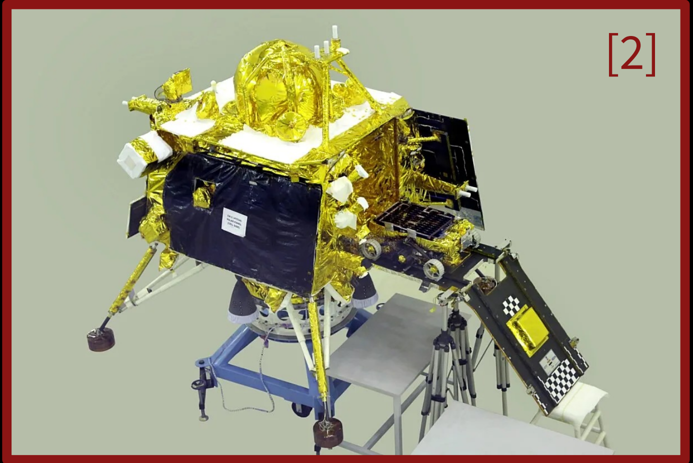
Kaila M. Y. Coimbra and Grace Gao

A New Era in Lunar Exploration

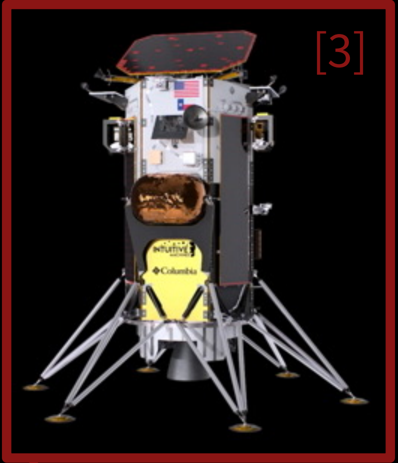
NAV LAB

*Dates are from launch to landing.
“Success” indicates successful landing.

Chandrayaan-3 (success)[2]
Jul 14 – Aug 23, 2023
ISRO’s lander and rover (IND)



Peregrine Mission 1 (failed)
Jan 8 – Jan 18, 2024
Astrobotic’s lunar landing mission, propellant leak (USA)

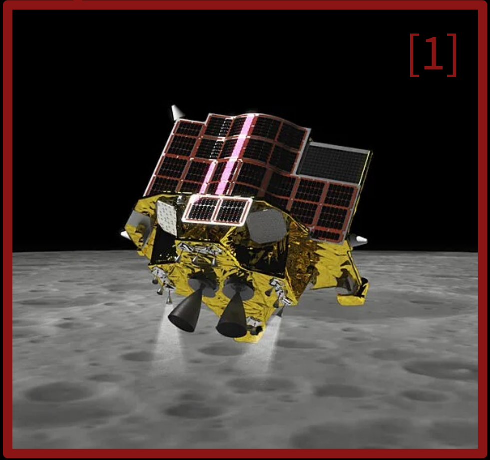


IM-1 (success)[3]
Feb 15 – Feb 22, 2024
Intuitive Machines’s Nova-C Odysseus lander (USA)

iSpace M1 (failed)
Dec 11, 2022 – Apr 25, 2023
iSpace’s lunar lander; crashed during landing (JPN)

Artemis I (success)
Nov 16 – Dec 11, 2022
NASA’s uncrewed test flight (USA)

Luna 25 (failed)
Aug 10 – Aug 19, 2023
Roscosmos’s lunar landing mission; crashed into surface (RUS)



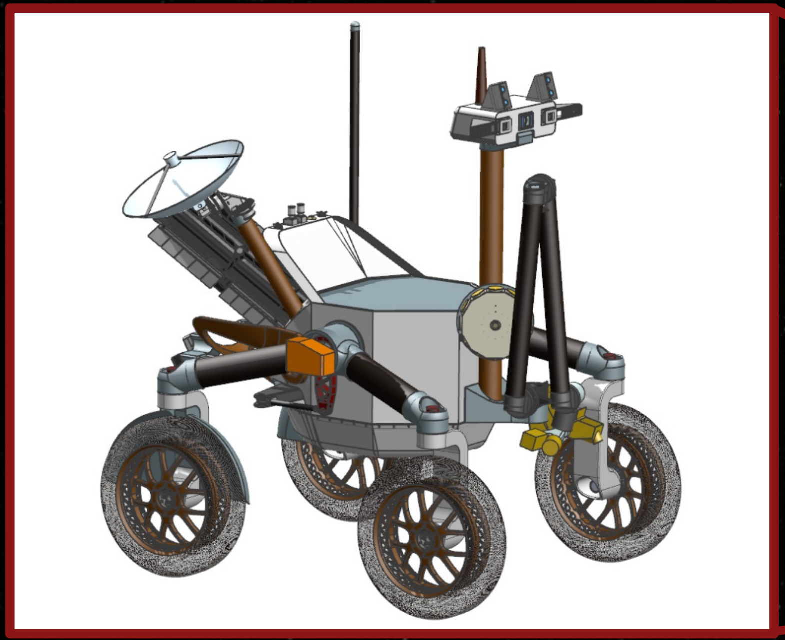
SLIM (success)
Sep 6, 2023 – Jan 19, 2024
JAXA’s precision lunar lander (JPN)

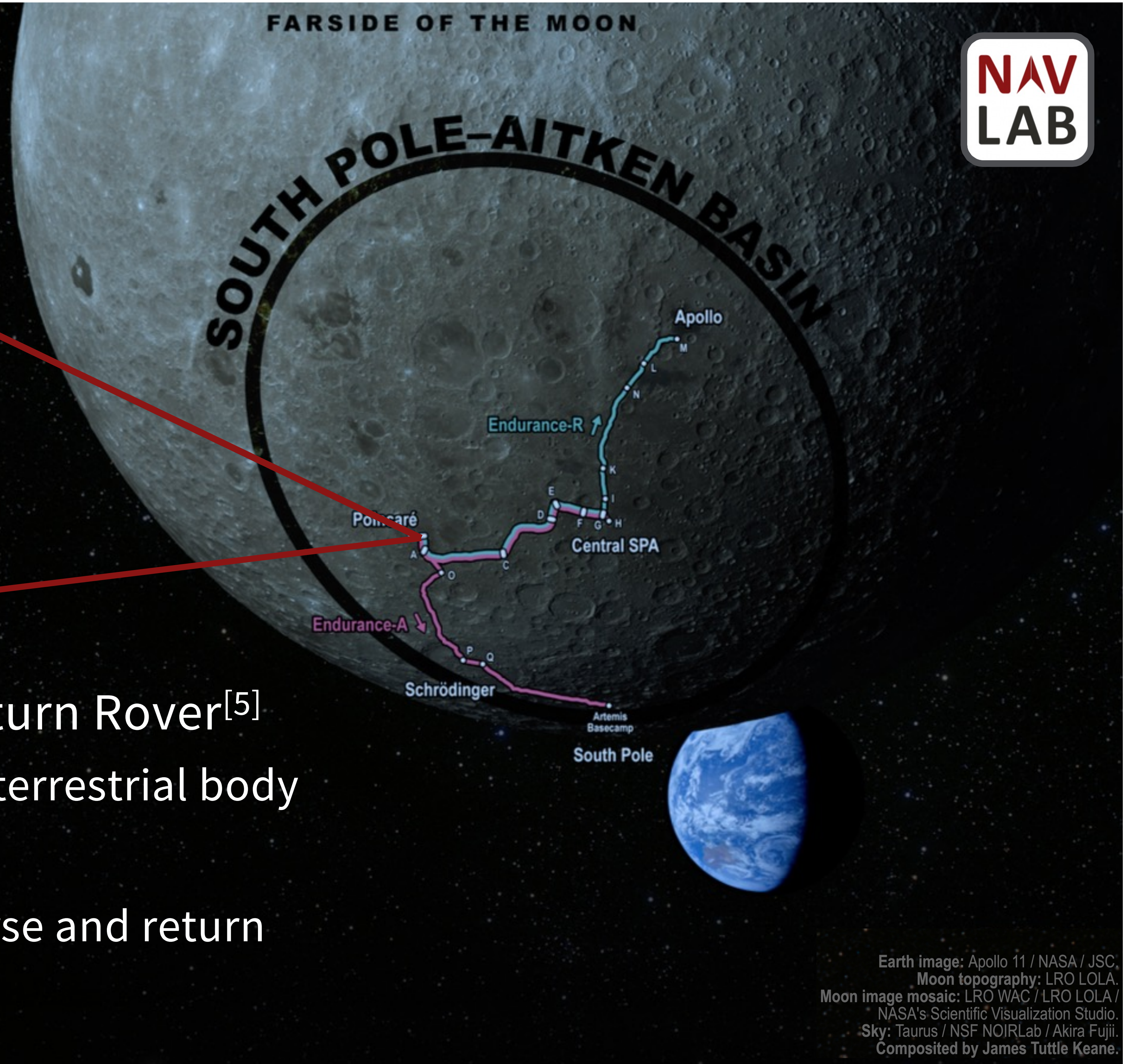
Chang’e-6 (success)[4]
May 3 – Jun 1, 2024
Brought back first samples from far side of the Moon (CHN)

DRO-A and DRO-B (failed)
Mar 13 – Mar 13, 2024
Pair of lunar navigation satellites, lost during launch (CHN)

[1] The Planetary Society, n.d.
[2] ISRO, 2023
[3] NASA, 2024
[4] The Planetary Society, n.d.

NASA Endurance Mission



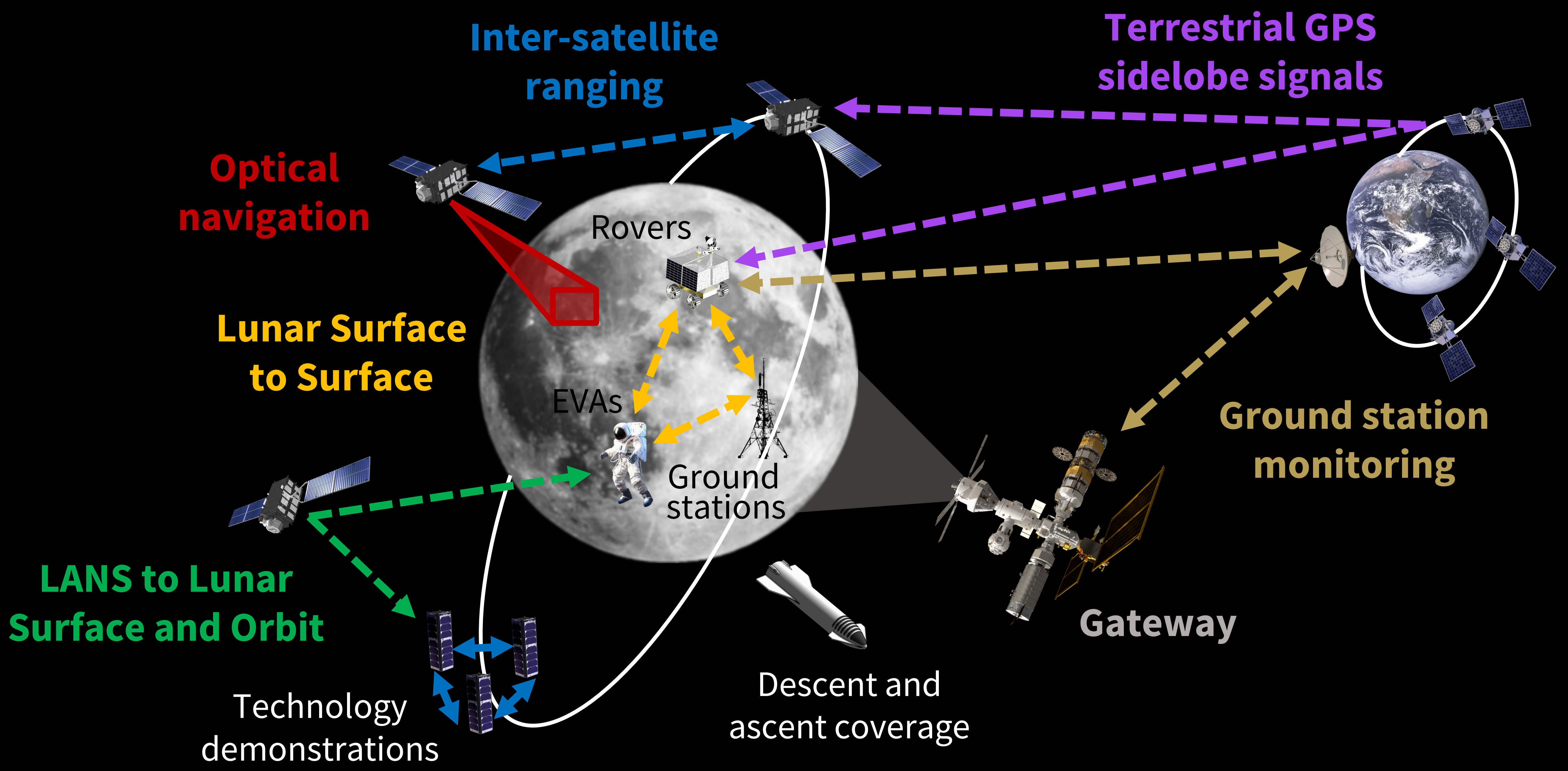


Lunar SPA Basin Traverse and Sample Return Rover[5]

- First rover to traverse 2000 km on an extraterrestrial body
- *Launch*: ~2032
- *Mission*: Collect 12 samples along its traverse and return samples to Artemis astronauts

[5] Keane et al., 2022

The Upcoming Lunar PNT Landscape



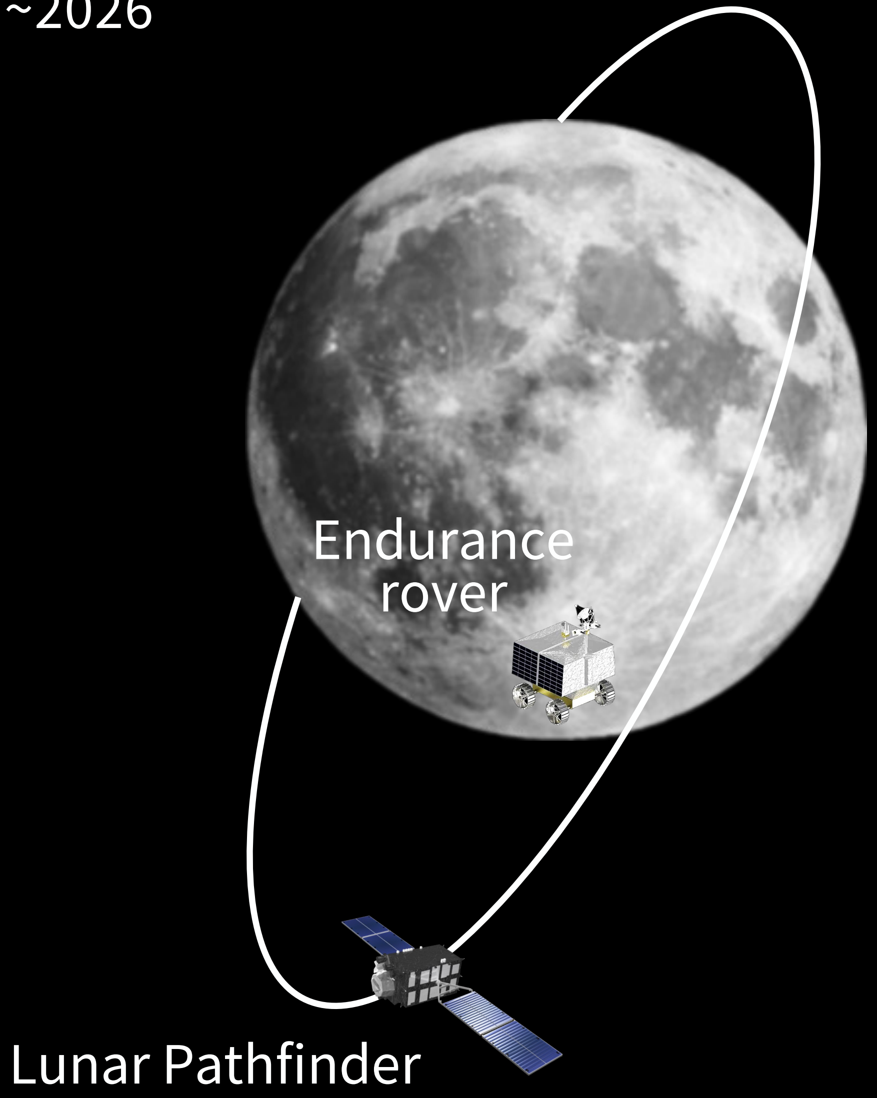
LANS – Lunar Augmented Navigation Service

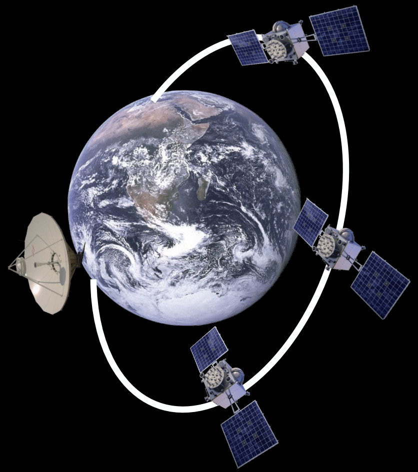
Referenced Schier, 2022.

Lunar Pathfinder

Data Relay Satellite in an elliptical lunar frozen orbit (ELFO)[6]

- *Mission*: Provide communication services to any lunar asset
- Operational starting ~2026



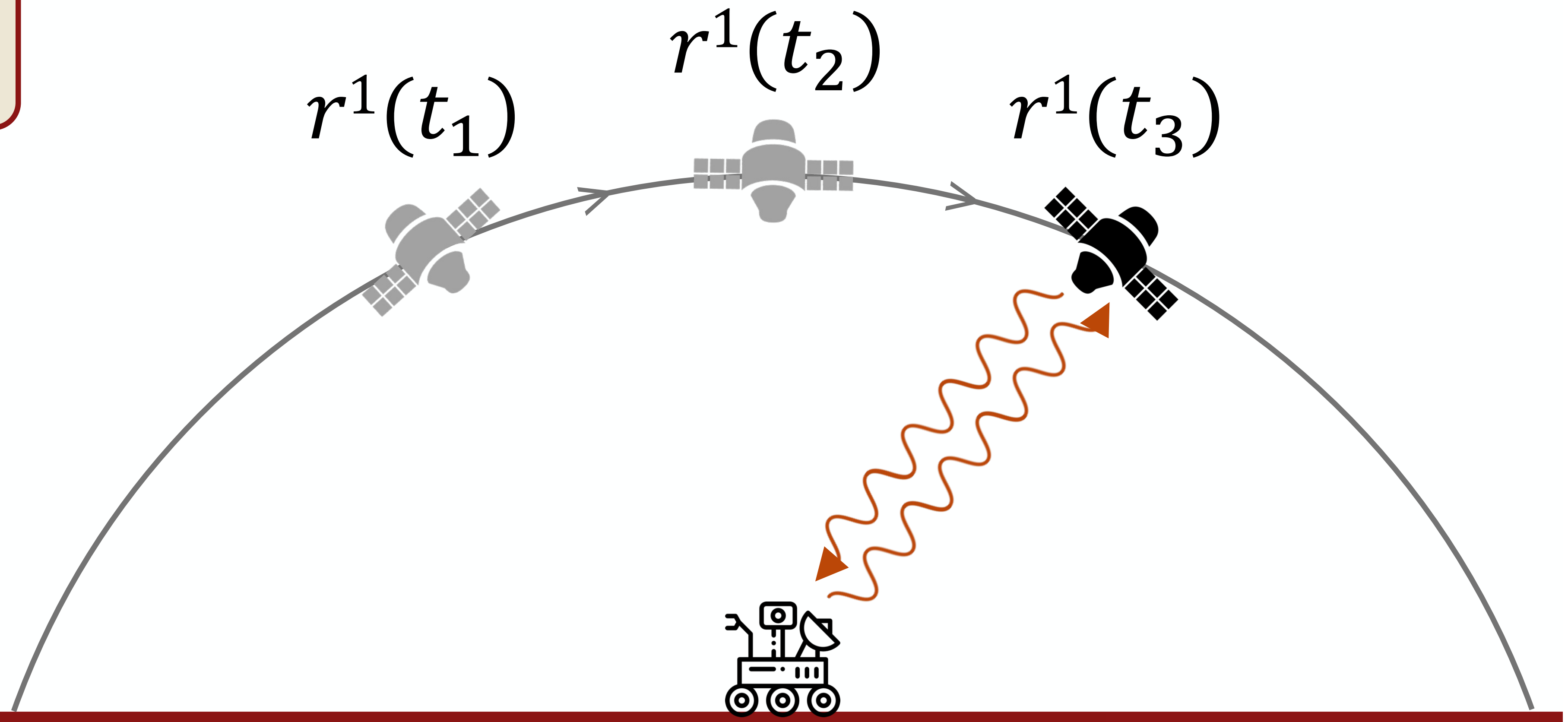


Prior Work: How to localize with only one satellite?

Approach: The rover, while stationary, accumulates navigation observables from a single satellite and refines its position estimate over time.

(1) Two-way ranging measurements[7]

- Rover needs to be stationary for up to 4 hours to achieve sub-10 m accuracy
- Requires a navigation payload



[7] Cortinovis et al., 2024

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(1) Two-way ranging measurements[7]

- Rover needs to be stationary for up to 4 hours to achieve sub-10 m accuracy
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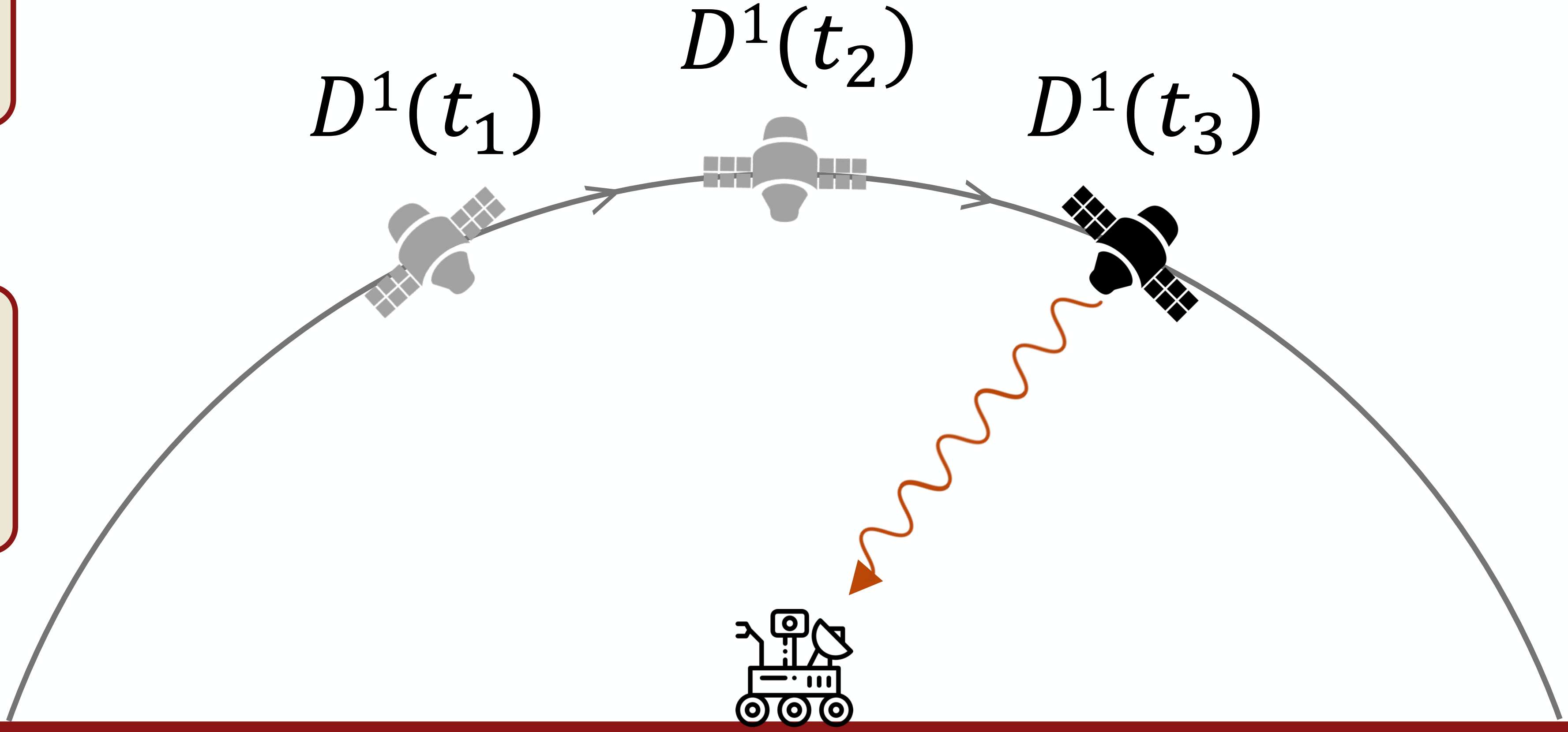
(2) Doppler shift measurements[8]

- Rover needs to be stationary for up to 16 hours to achieve sub-10 m accuracy
- Does not require a navigation payload



[7] Cortinovis et al., 2024
[8] Coimbra et al., 2024

Problem Scenario: Introducing a second satellite

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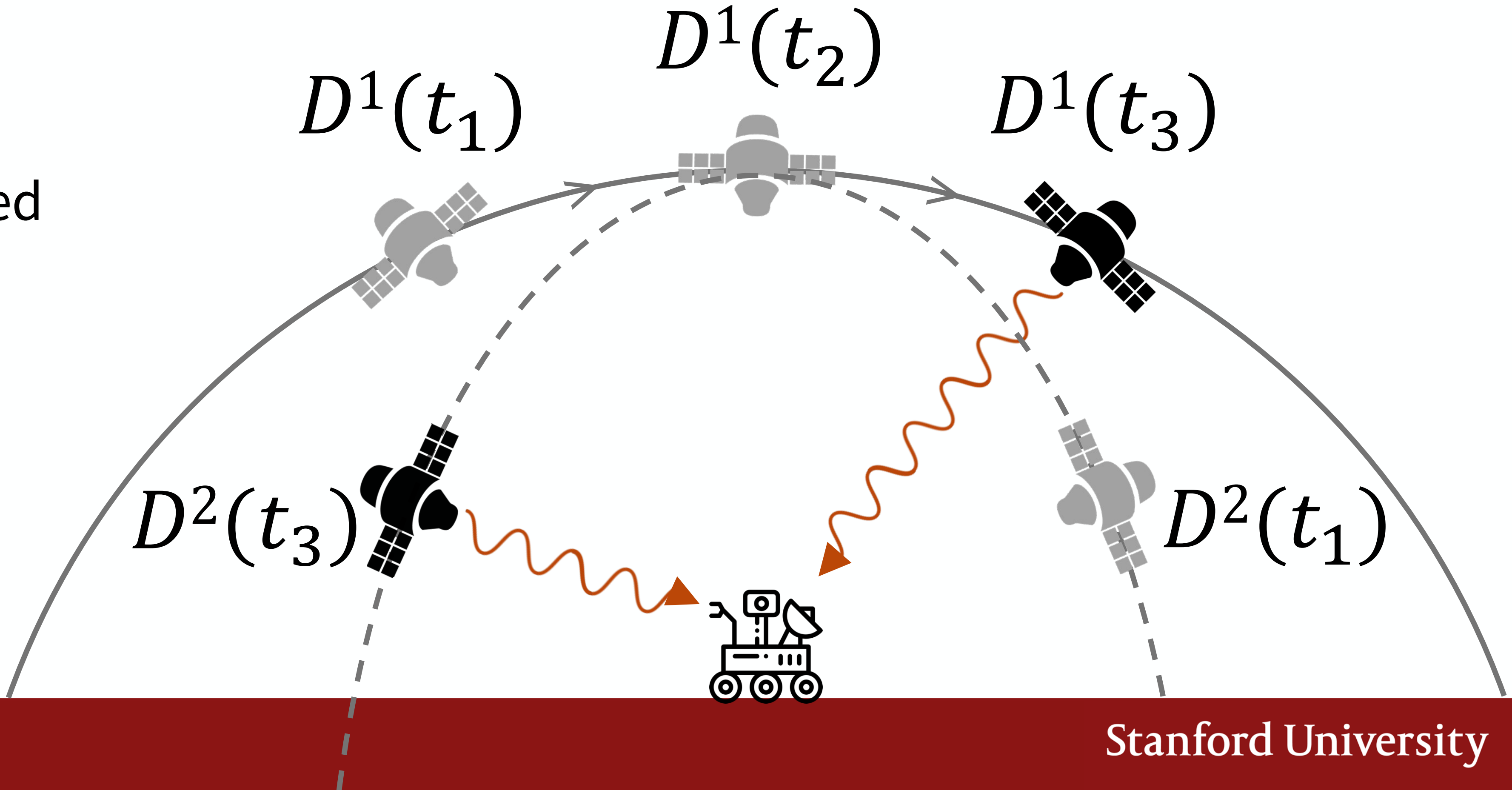
Key Questions:

(1) What will the orbit of the auxiliary satellite have to look like to best complement the Lunar Pathfinder's coverage?

(2) What *improvement* in the rover's localization accuracy can be achieved by introducing a second satellite?



Overview

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Measurement Modeling and Filtering

- Simulate and filter realistic measurements from the satellites' communication signals to obtain a positioning fix.

Orbit Design for Auxiliary Satellite

- Determine the orbital elements for the auxiliary satellite that maximizes coverage of the lunar South Pole region.

State Estimation Performance and Comparison

- Evaluate the rover's absolute localization accuracy using two non-navigation satellites and compare to the single-satellite scenario.

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Measurement and Error Modeling

Determine pseudorange rate measurements from Doppler shift.

$$\dot{\rho} = -\frac{Dc}{f_C}$$

➡

In simulation, generate pseudorange rate measurements that the rover will observe.

$$\tilde{\dot{\rho}}(t) = \underbrace{\mathbf{v}_{\boldsymbol{s}}(t) \cdot \frac{\mathbf{r}_{\boldsymbol{s}}(t) - \mathbf{r}_{\boldsymbol{r}}}{\|\mathbf{r}_{\boldsymbol{s}}(t) - \mathbf{r}_{\boldsymbol{r}}\|}}_{\text{True range rate}} + \underbrace{c\left(\dot{\delta t}_r - \dot{\delta t}_s\right)}_{\text{Relative clock drift}} + \underbrace{\epsilon_{\dot{\rho}}}_{\text{Error}}$$

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Measurement Errors

$$\epsilon_{\dot{\rho}} \sim \mathcal{N}(0, \sigma_{\dot{\rho}}^2)$$

Contributions include thermal and phase noise (phase scintillation is negligible).

$$\sigma_{\dot{\rho}}^2 = \sigma_t^2 + \sigma_c^2$$

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$$\sigma_{\dot{\rho}}^2 = \sigma_t^2 + \sigma_c^2$$

Satellite Ephemeris Errors

The rover has erroneous knowledge of the satellite's position and velocity when predicting its expected measurements for filtering.

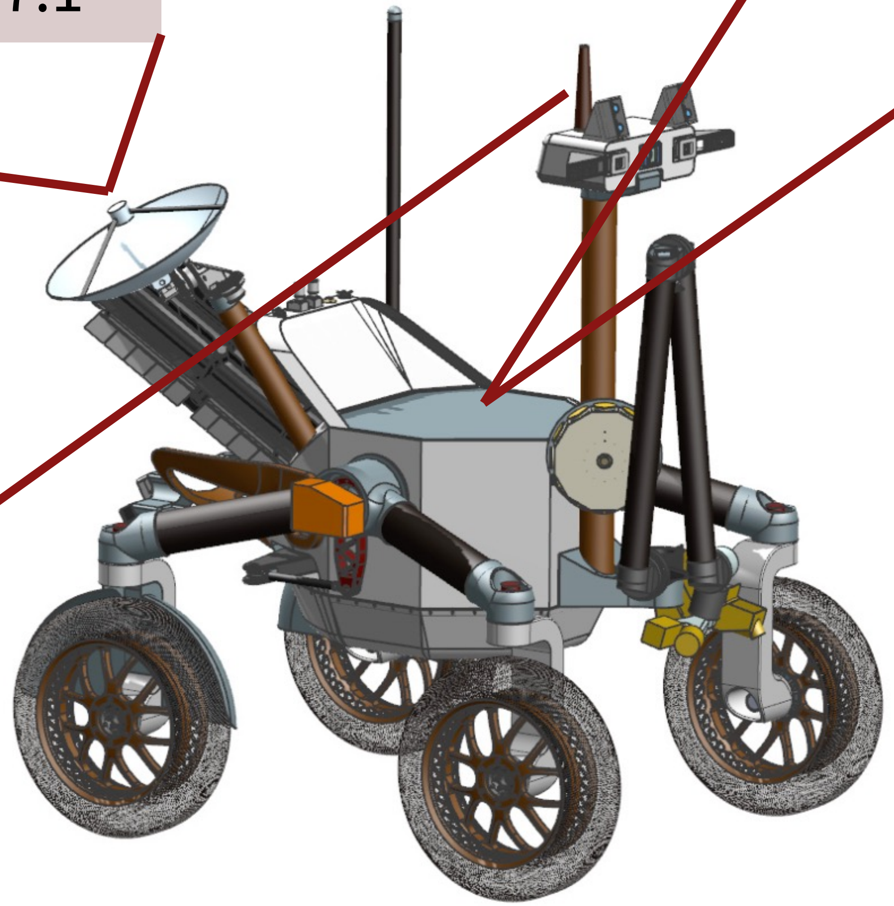
$$\tilde{\mathbf{r}}_\mathbf{s} = \mathbf{r_s} + \epsilon_{e,p} \quad , \quad \epsilon_{e,p} \sim \mathcal{N}(0, \sigma_{e,p_{xyz}}^2 \mathbf{I_{3\times3}})$$

$$\tilde{\mathbf{v}}_\mathbf{s} = \mathbf{v_s} + \epsilon_{e,v} \quad , \quad \epsilon_{e,v} \sim \mathcal{N}(0, \sigma_{e,v_{xyz}}^2 \mathbf{I_{3\times3}})$$

Endurance Rover's Relevant Components

75 cm high gain omnidirectional antenna[5]

S-band frequency[4]	2050 Hz
EIRP @ HPBW[4]	26.5 dBW @ 7.1 °



Low gain antenna

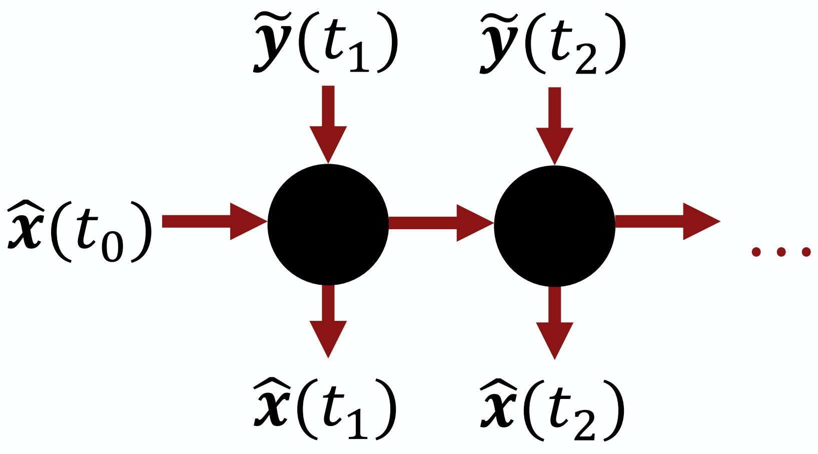
Stanford Research Systems PRS 10 clock[9, 10]

Size	155 cm^3
Weight	0.6 kg
Power	14.4 W
TDEV per day	0.07 µs

[5] Keane et al., 2022
[9] Schmittberger et al., 2020.
[10] Bhamidipati, et al., 2022.

State Estimation Considerations

Extended Kalman Filter



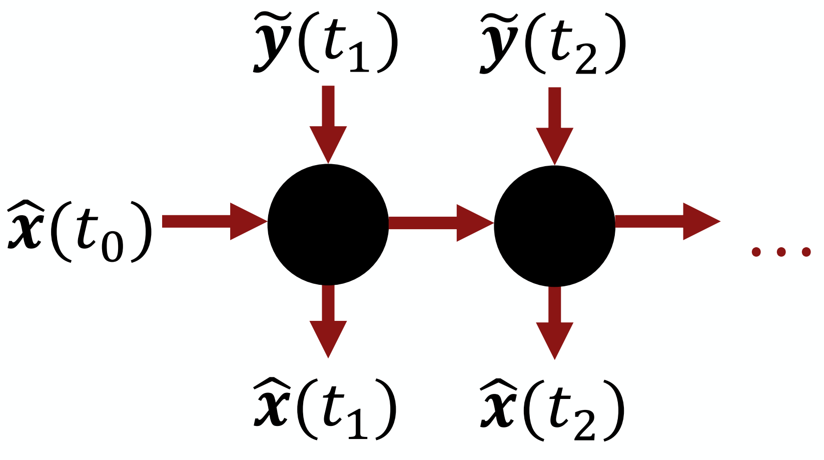
EKF processes one measurement at each time step.

- Not desirable for system with limited observability
- Consider filter that processes multiple measurements over time

	Rover state	Measurements
Observed	$\tilde{\boldsymbol{x}}$	$\tilde{\boldsymbol{y}}$
Expected	$\hat{\boldsymbol{x}}$	$\hat{\boldsymbol{y}}$

State Estimation Considerations

Extended Kalman Filter



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Observed	$\tilde{\boldsymbol{x}}$	$\tilde{\boldsymbol{y}}$
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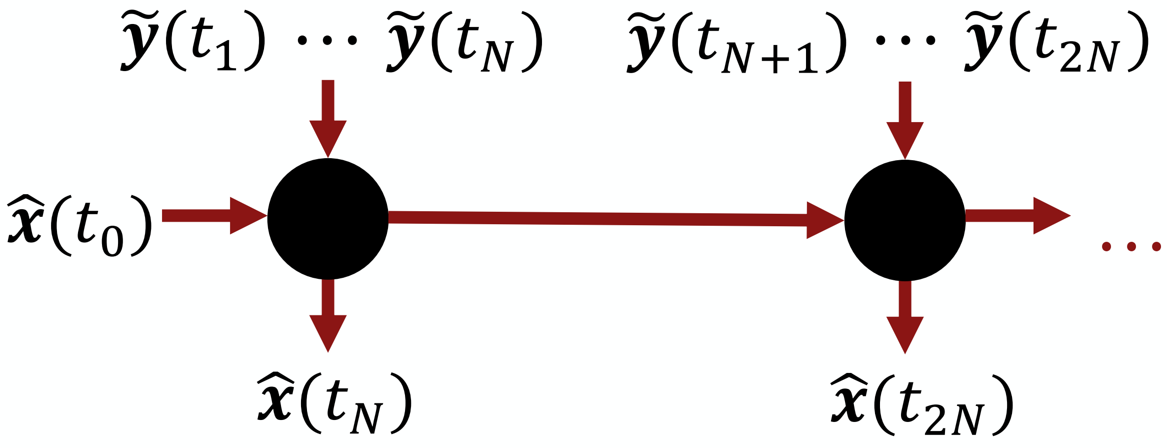
Weighted Batch Filter



Minimize $\quad C = \|\tilde{\mathbf{y}} - \hat{\mathbf{y}}_{k+1}\|^2_{\mathbf{W}}$

$$\begin{cases} \mathbf{W} = \text{diag}\left(\boldsymbol{\sigma}^{-2}_{tot,1} \;, \; \boldsymbol{\sigma}^{-2}_{tot,2}\right) \\ \sigma^2_{tot} = \underbrace{\sigma^2_{e,v}}_{\text{Eph. vel. error}} + \underbrace{\sigma^2_{\dot{\rho}}}_{\text{Meas. error}} \end{cases}$$

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- Determine the orbital elements for the auxiliary satellite that maximizes coverage of the lunar South Pole region.

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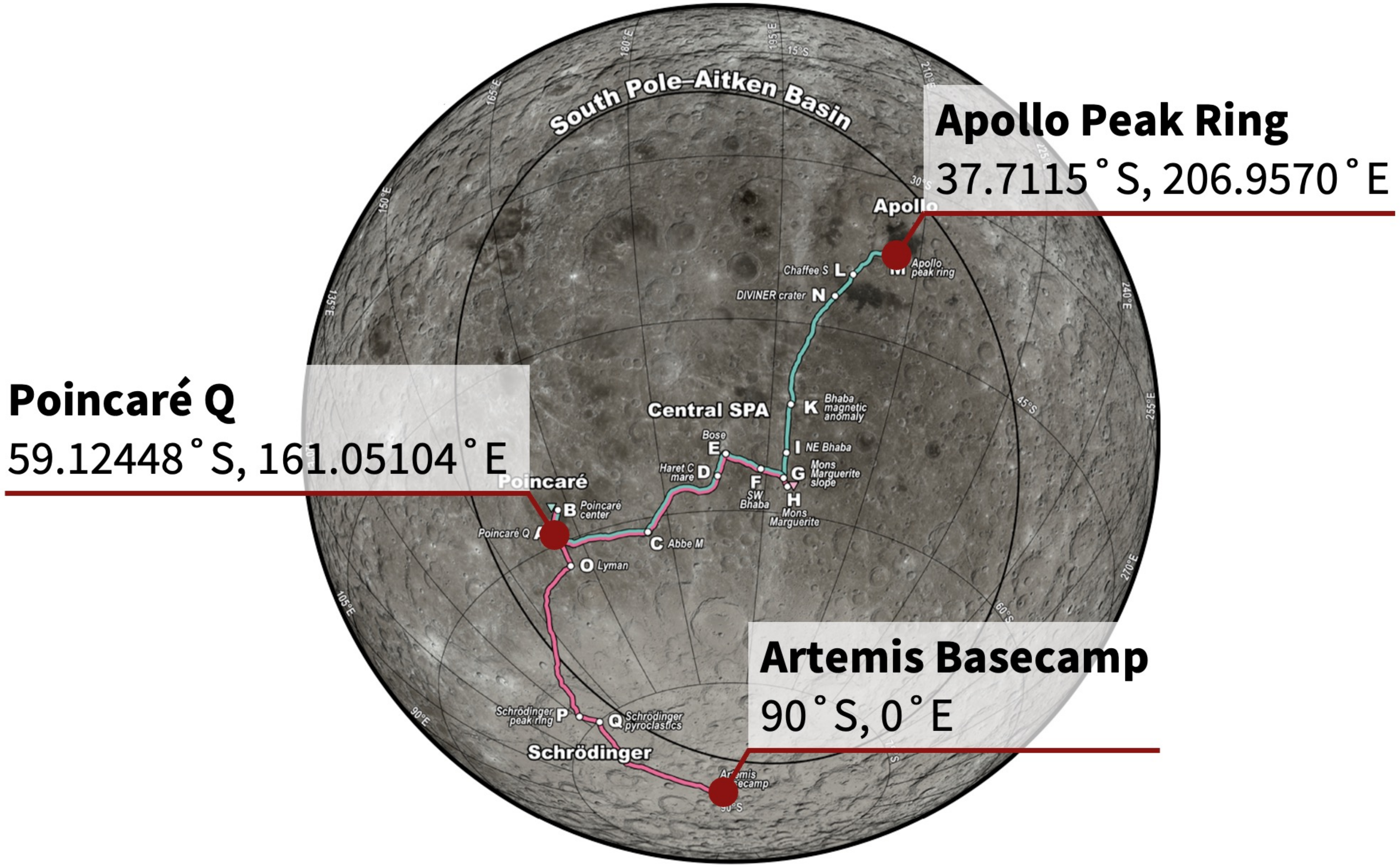
Rover Positioning Assumptions and Test Sites

- Assume that the rover is stationary during the measurement accumulation window
- Set 3D initial rover positioning error to be 100 m
- Choose three key waypoints as simulation testing locations



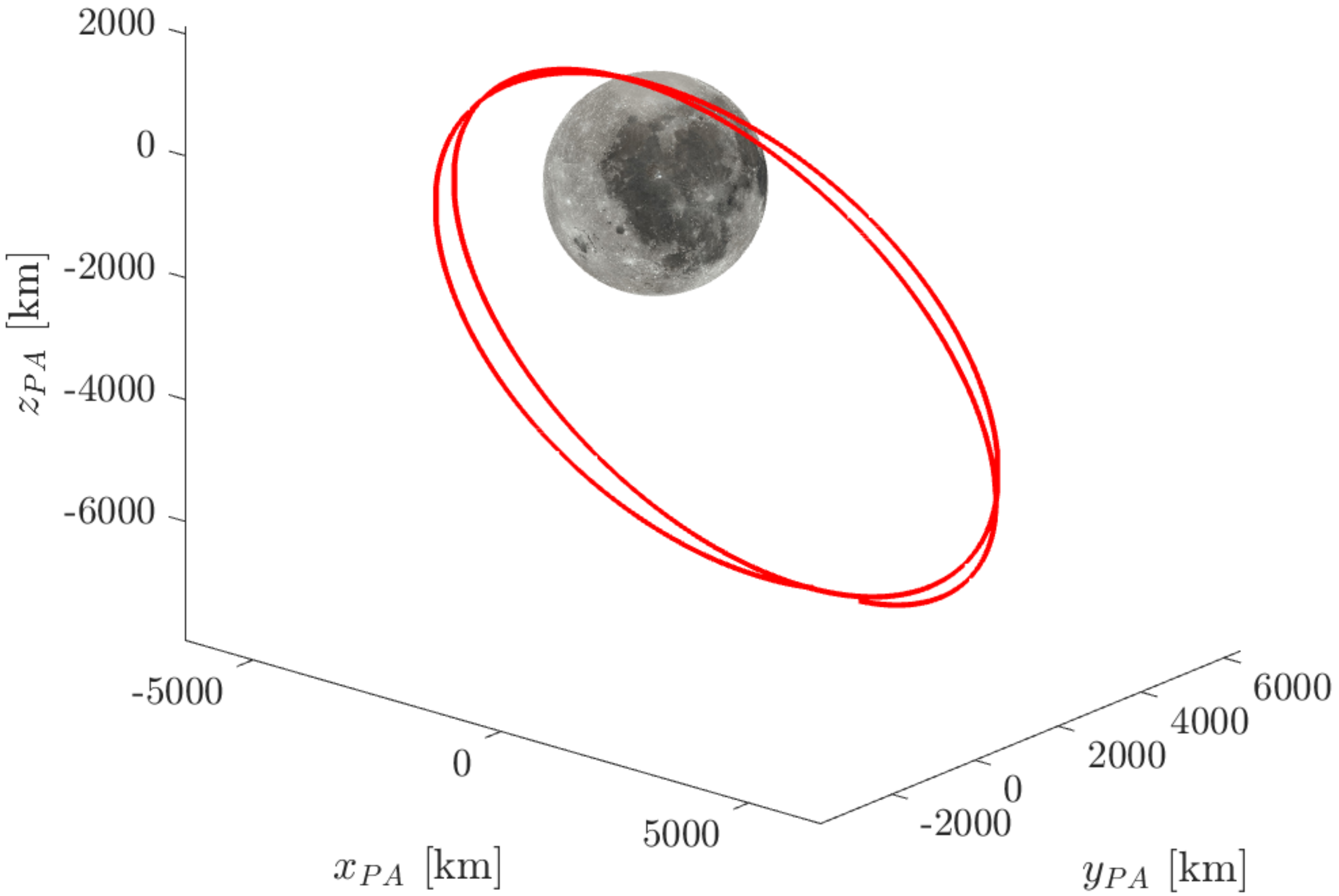
Chosen locations along rover's traverse[5]

[5] Keane et al., 2022

Lunar Pathfinder Orbit Model

Simulation Parameters	
LP orbital period	10.84 hours
Initial epoch	2030/10/01 00:00:00 UTC

Lunar Pathfinder's Orbital Elements[6]	
Semi-major axis a	5740 km
Eccentricity e	0.58
Inclination i	54.856 $^\circ$
RAAN Ω	0 $^\circ$
Argument of the Periapsis ω	86.322 $^\circ$
Mean Anomaly M	180 $^\circ$



Two orbital periods of the Lunar Pathfinder in the fixed Moon Principal Axis frame of reference

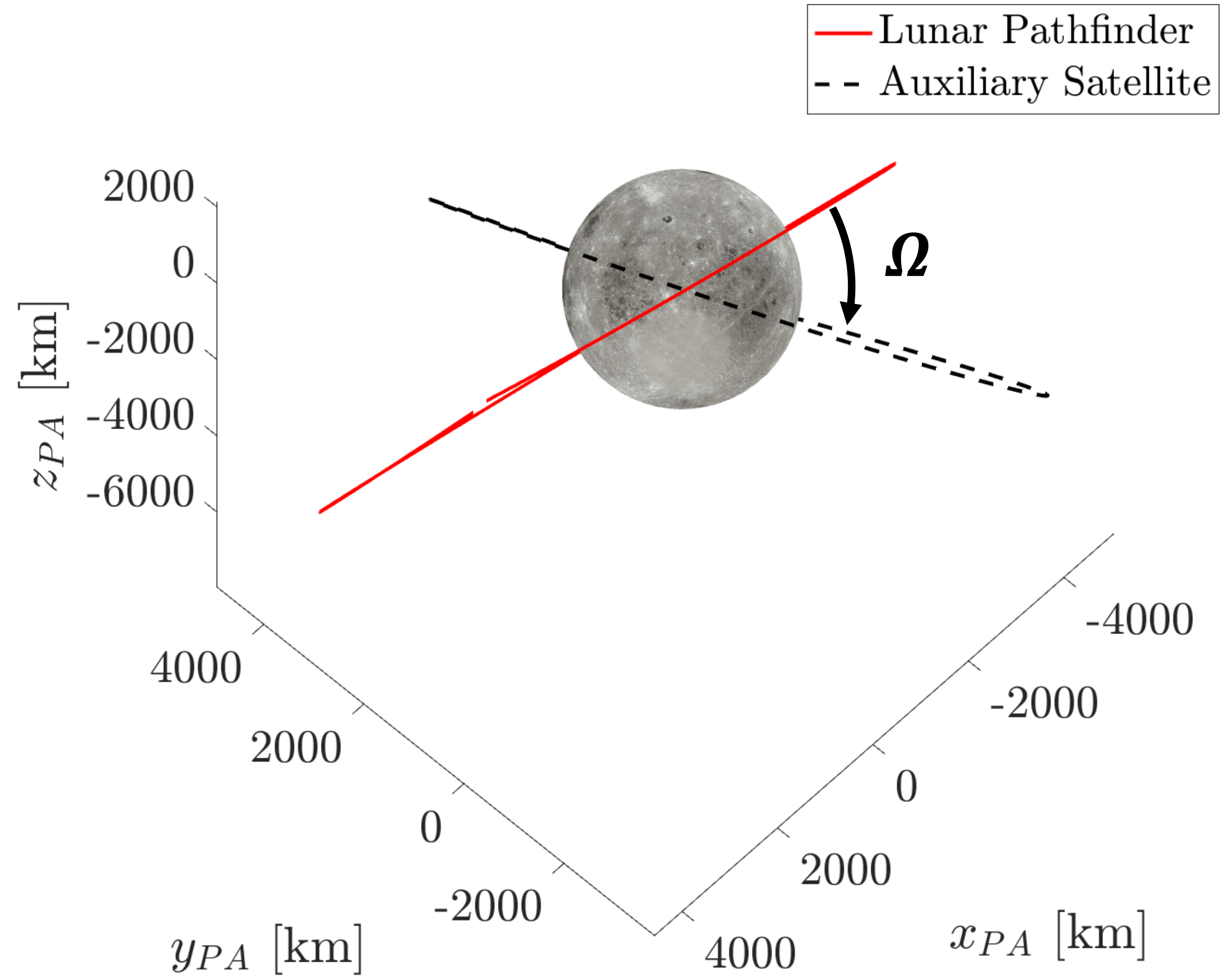
[6] SSTL, 2022

Design Constraints for the Auxiliary Satellite

Goal: Maintain long-duration coverage of the Lunar South Pole region → ELFO

- Freeze a, e, i, ω
- Tune RAAN: Minimize PDOP

Auxiliary Satellite's Orbital Elements	
Semi-major axis a	5740 km
Eccentricity e	0.58
Inclination i	54.856°
RAAN Ω	**30° to 240°**
Argument of the Periapsis ω	86.322°
Mean Anomaly M	180°



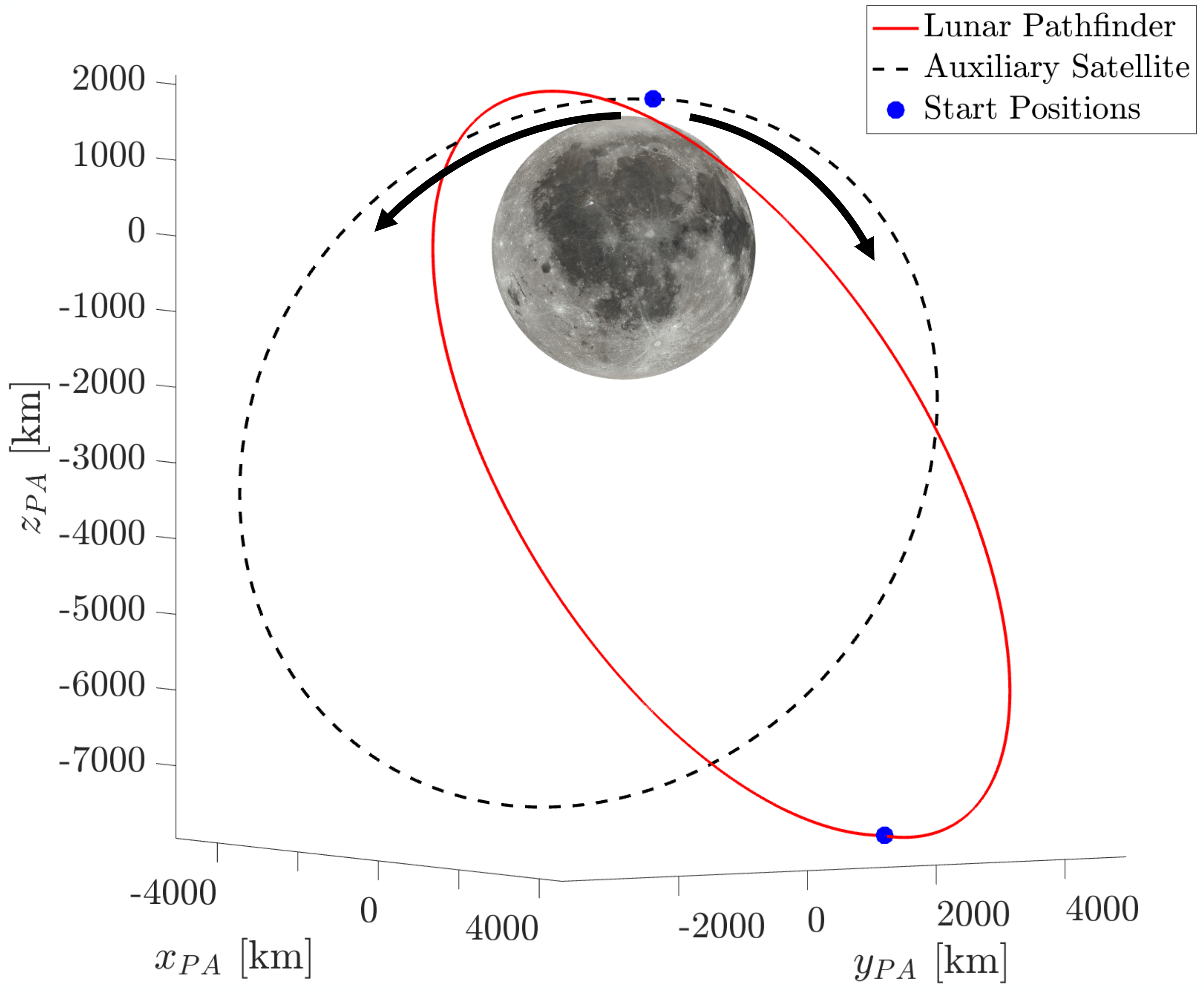
Orbits of Lunar Pathfinder and auxiliary satellite in the Moon PA frame (bottom view)

Design Constraints for the Auxiliary Satellite

Goal: Maintain long-duration coverage of the Lunar South Pole region → ELFO

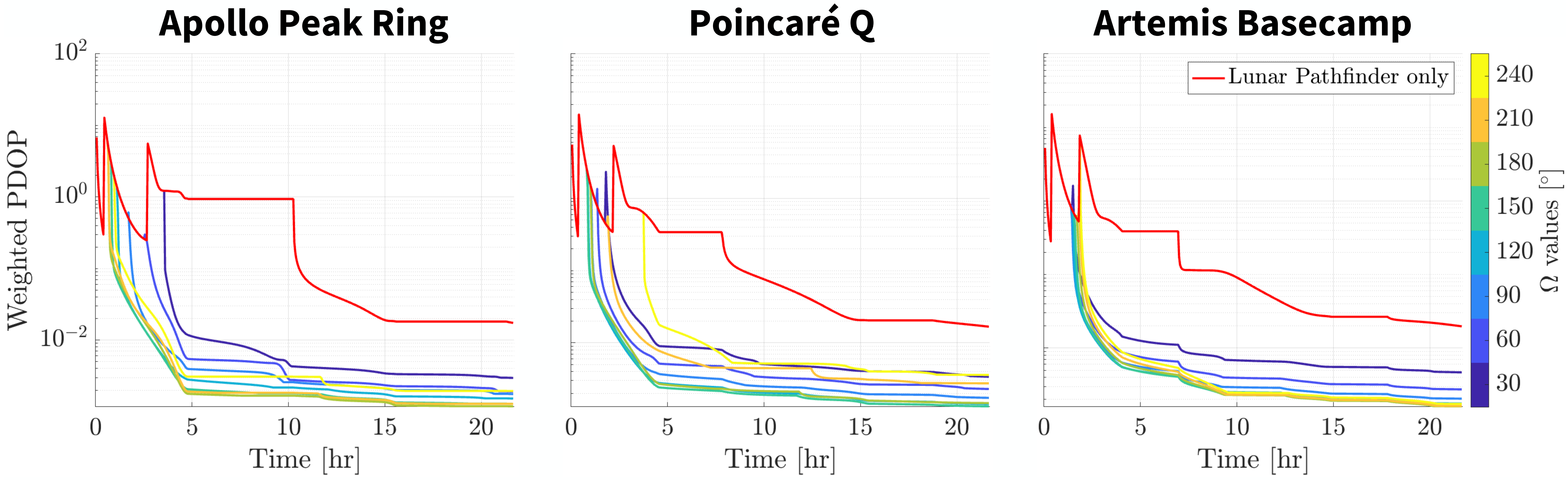
- Freeze a, e, i, ω
- Tune RAAN: Minimize PDOP
- Tune mean anomaly: Maximize visibility

Auxiliary Satellite's Orbital Elements	
Semi-major axis a	5740 km
Eccentricity e	0.58
Inclination i	54.856°
RAAN Ω	**30° to 240°**
Argument of the Periapsis ω	86.322°
Mean Anomaly M	**-90° (270°) to 90°**



Mean anomaly offset with respect to Lunar Pathfinder's mean anomaly of 180°

Auxiliary Satellite Orbit Selection: RAAN Ω



It is not immediately obvious which RAAN value corresponds to the lowest weighted PDOP (WPDOP) across all three locations over time.

Auxiliary Satellite Orbit Selection: RAAN Ω

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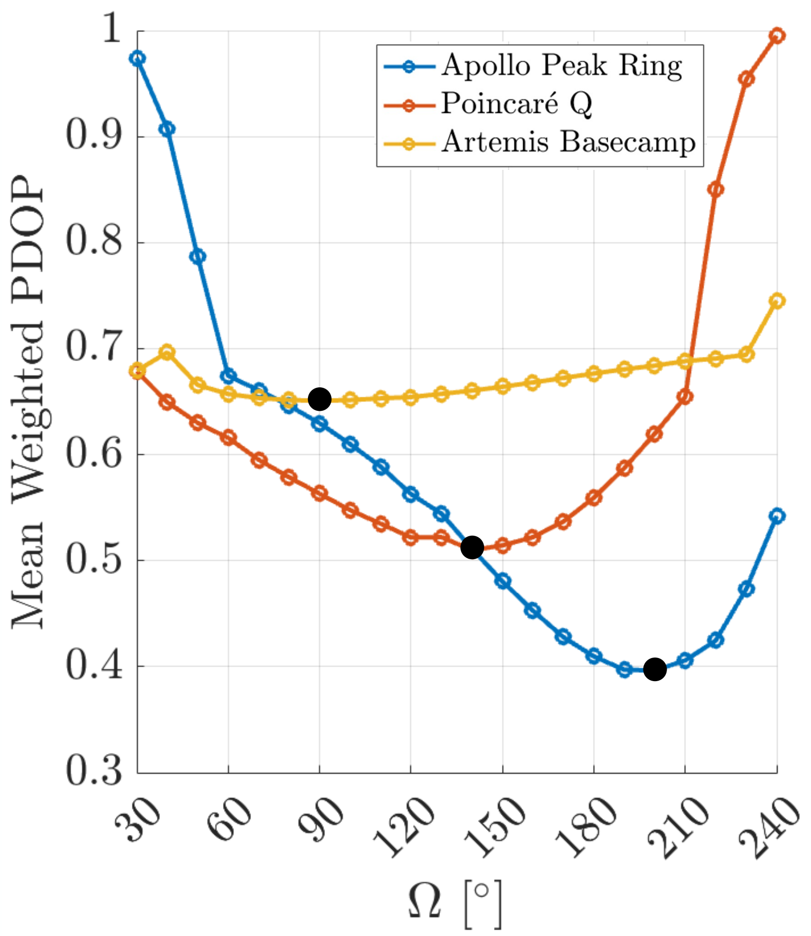
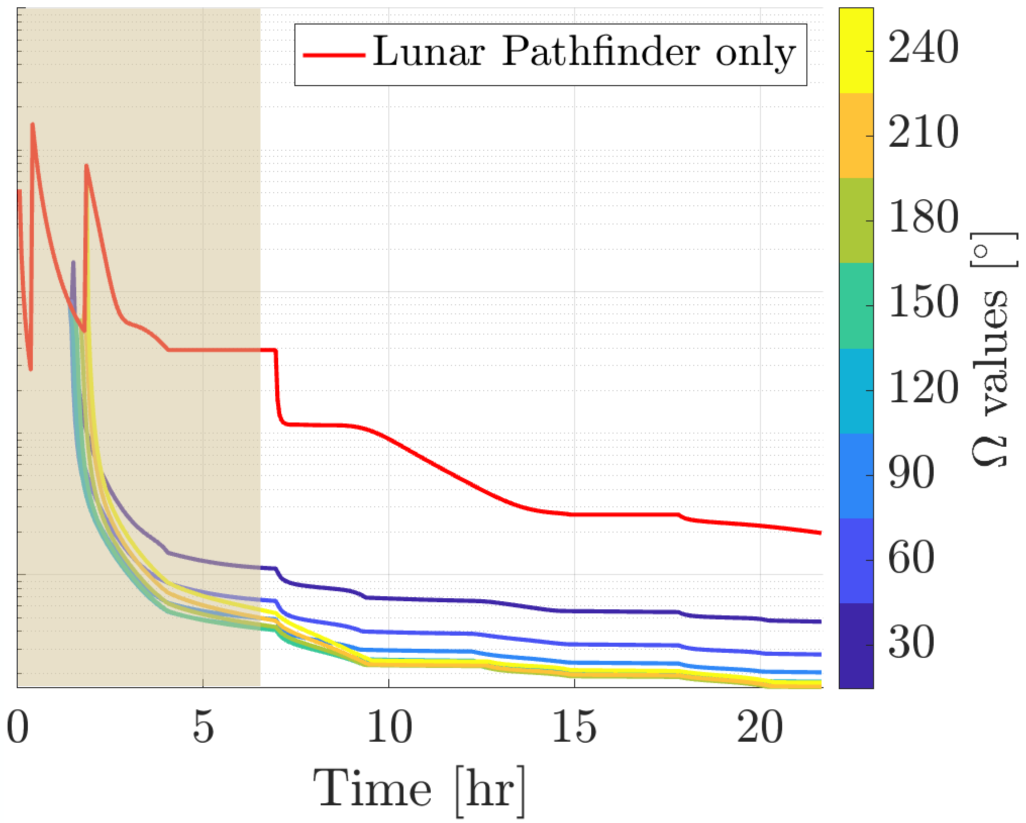
0 - 7 hours

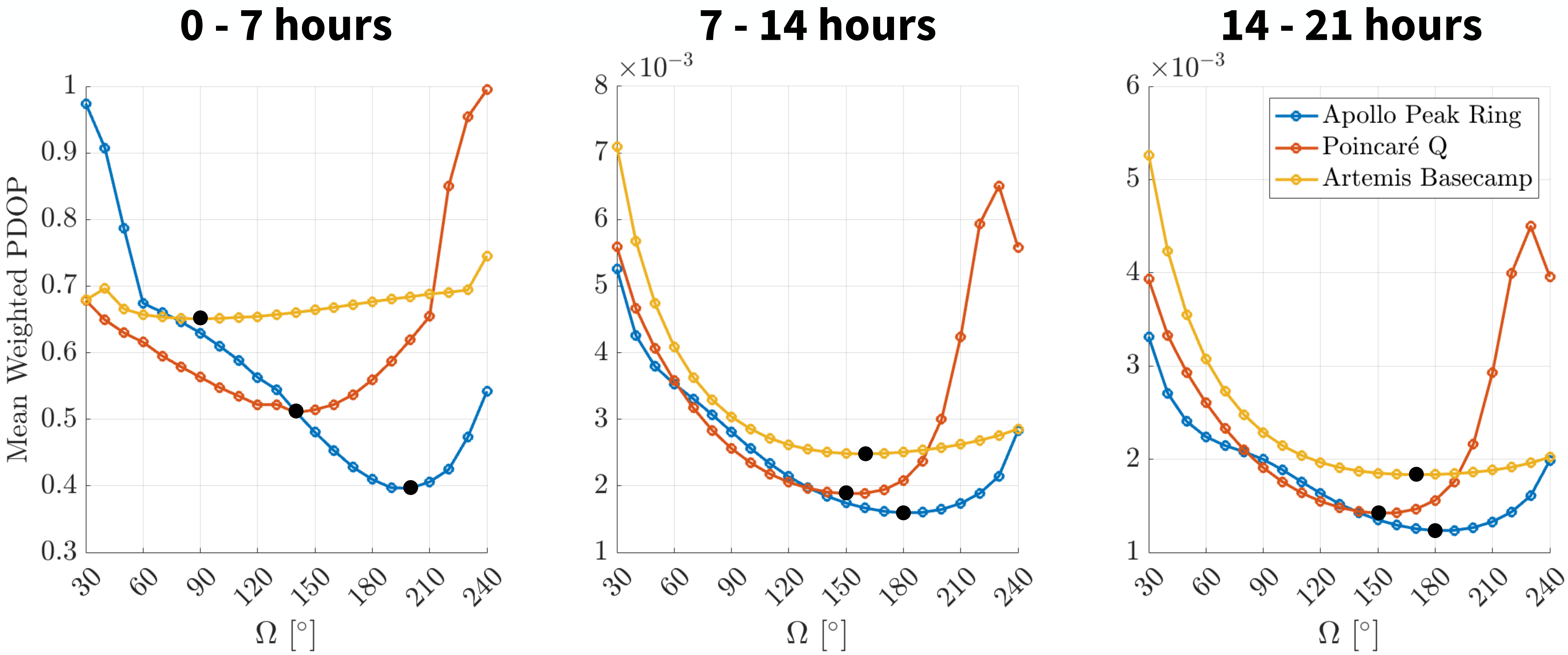


Compute block average over smaller time segments.

Artemis Basecamp

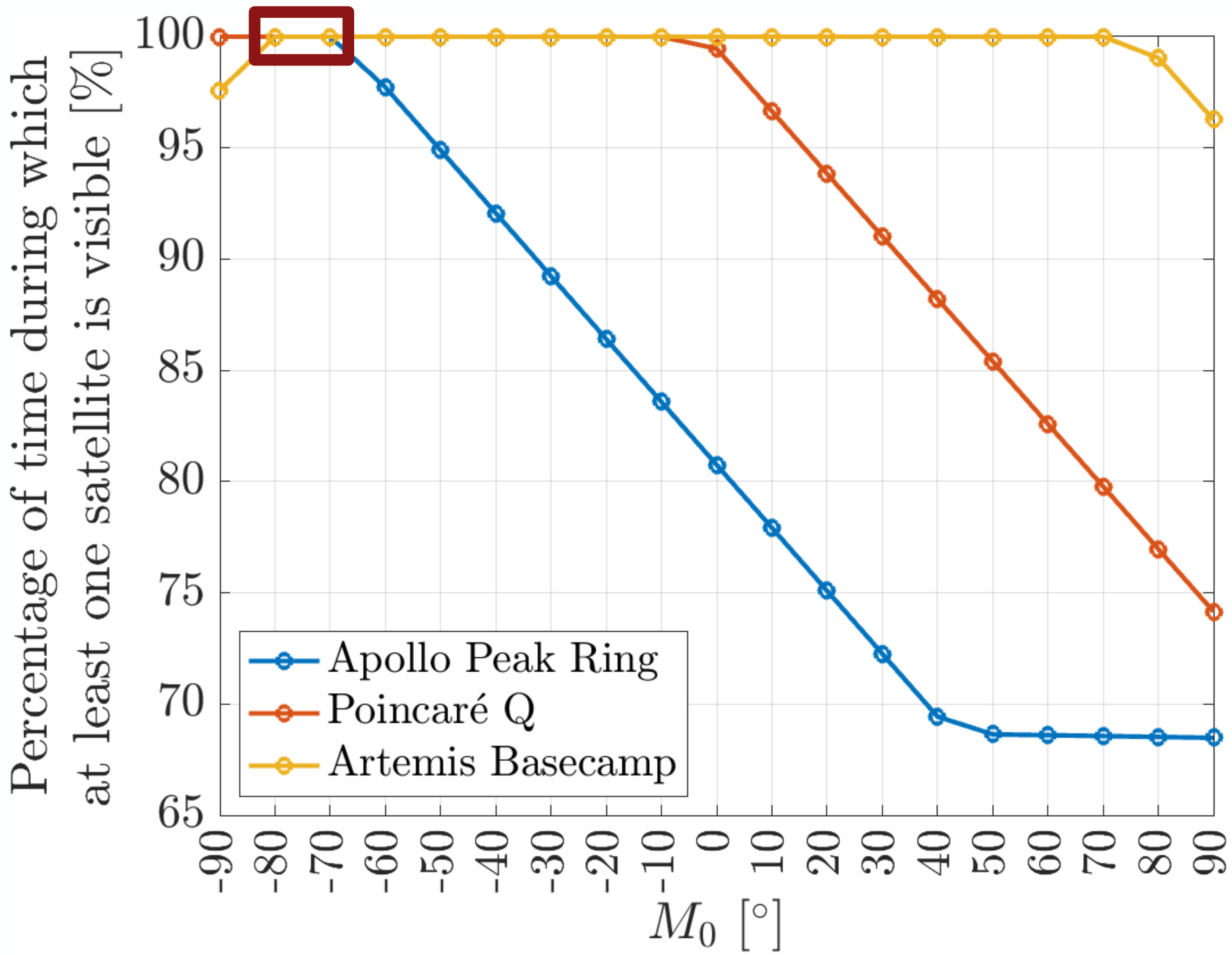


Auxiliary Satellite Orbit Selection: RAAN Ω



By averaging the lowest WPDOP per time segment across all three locations, the optimal RAAN value is approximately 160˚.

Auxiliary Satellite Orbit Selection: Mean Anomaly M_0

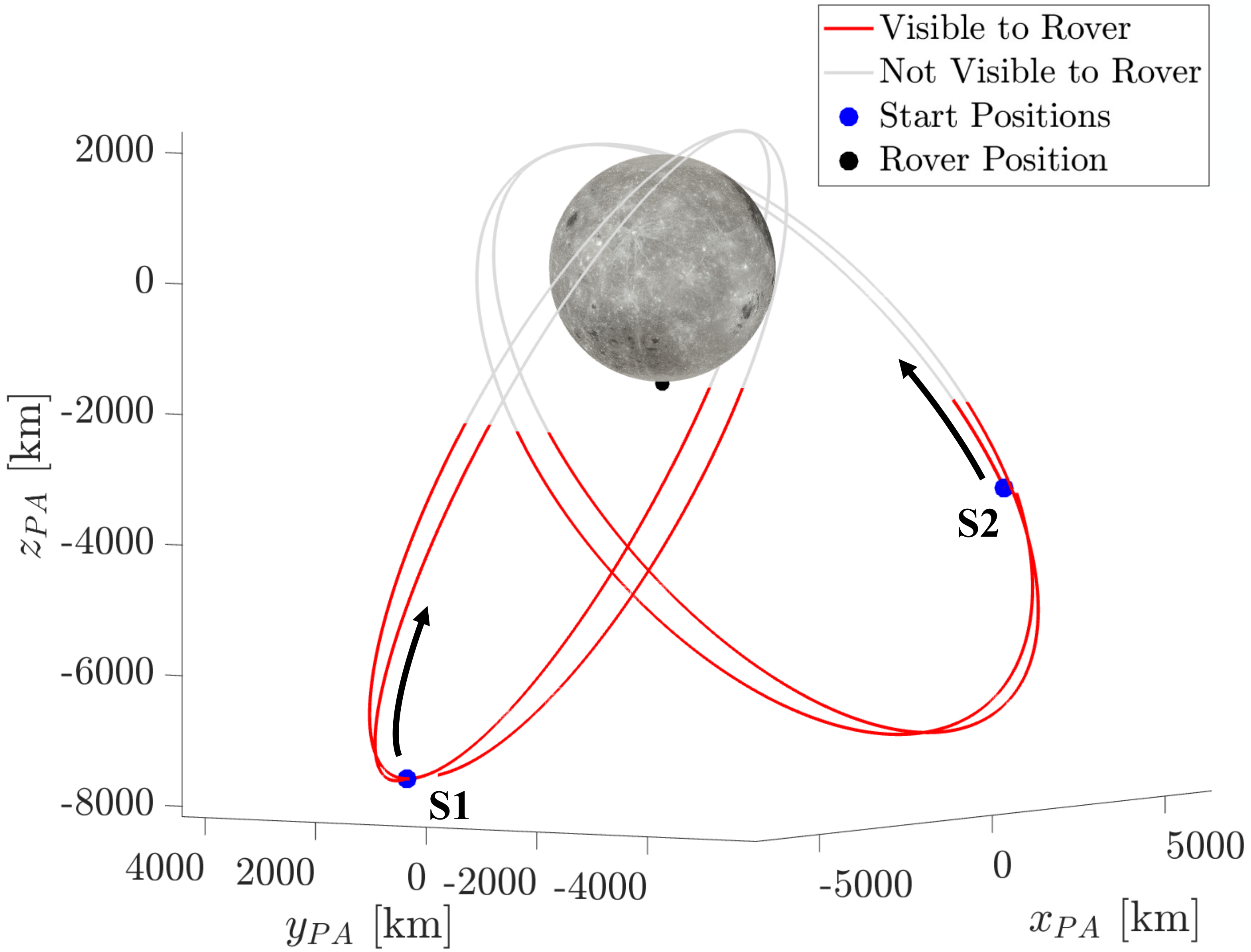


Visibility:

- Elevation mask > 5°
- C/N_0 > 30 dB Hz

To ensure continuous visibility, the mean anomaly of the auxiliary satellite should be -70° (equivalent to 290°), which is a 250° mean anomaly offset relative to the Lunar Pathfinder.

Auxiliary Satellite Orbit Selection



Orbital Elements	Lunar Pathfinder (S1)	Auxiliary Satellite (S2)
Semi-major axis	5740 km	5740 km
Eccentricity	0.58	0.58
Inclination	54.856 °	54.856 °
RAAN	**0 °**	**160 °**
Argument of the Periapsis	86.322 °	86.322 °
Mean Anomaly	**180 °**	**290 °**

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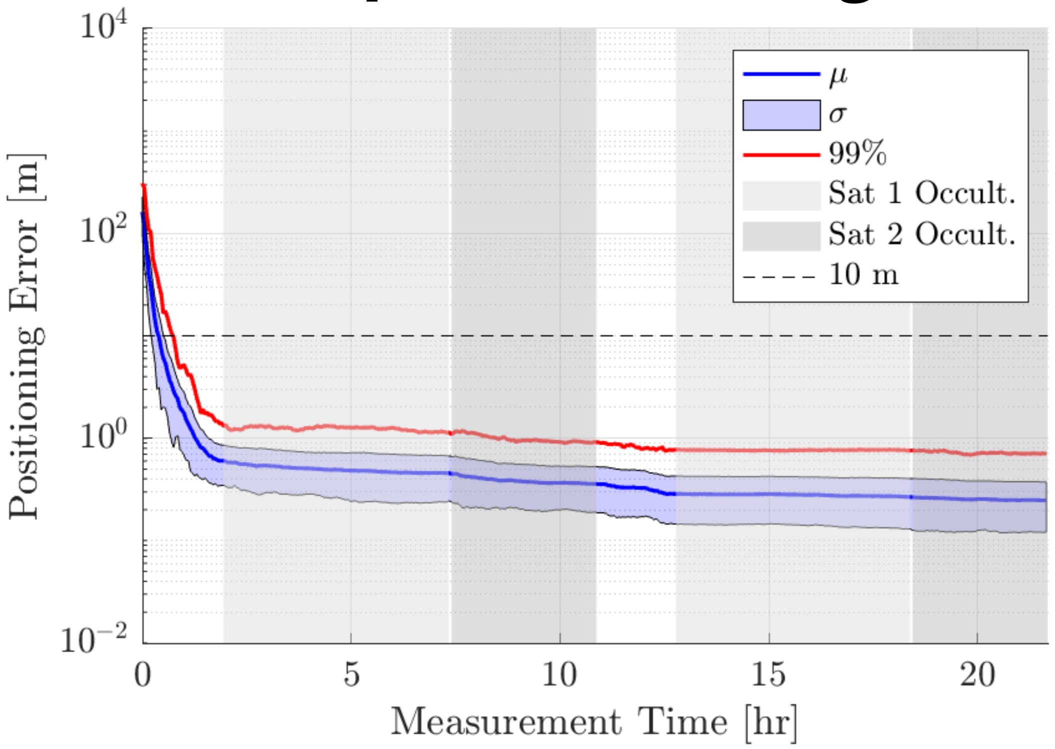
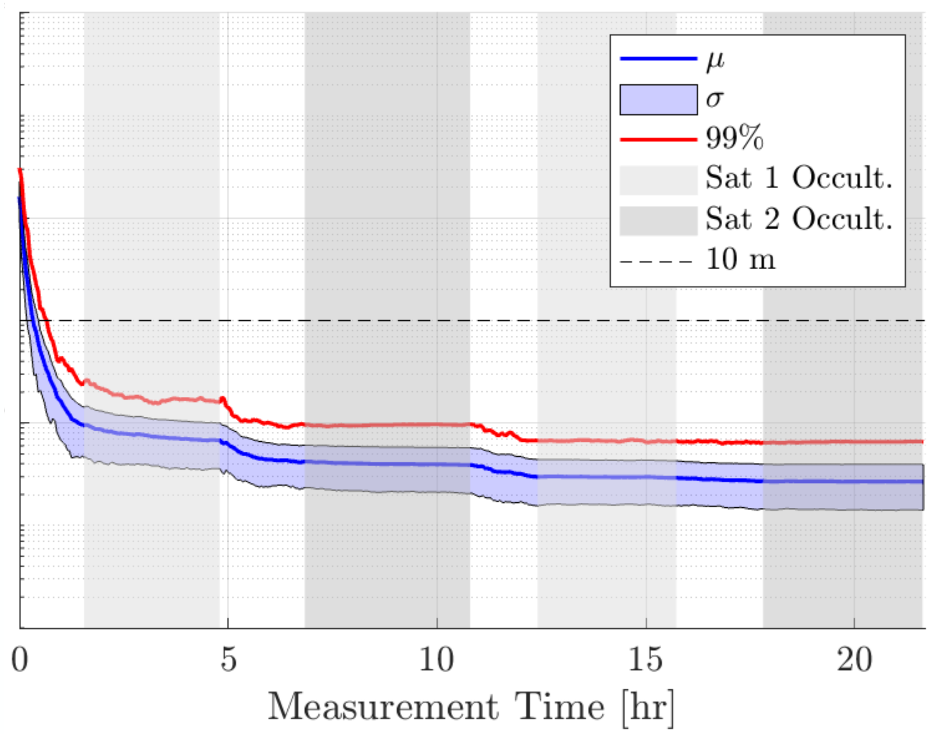
Positioning Error across 100 Monte Carlo Runs

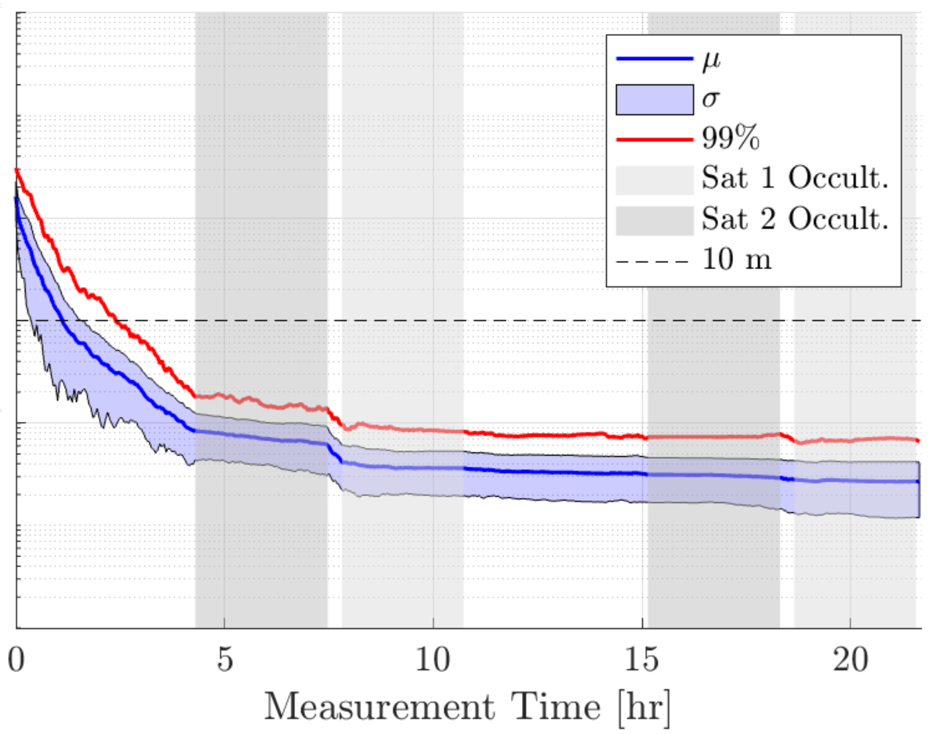
Apollo Peak Ring



Poincaré Q



Artemis Basecamp



Location	Mean [hr]	99% perc. [hr]
Apollo Peak Ring	0.39	0.73
Poincaré Q	0.33	0.65
Artemis Basecamp	1.11	2.39

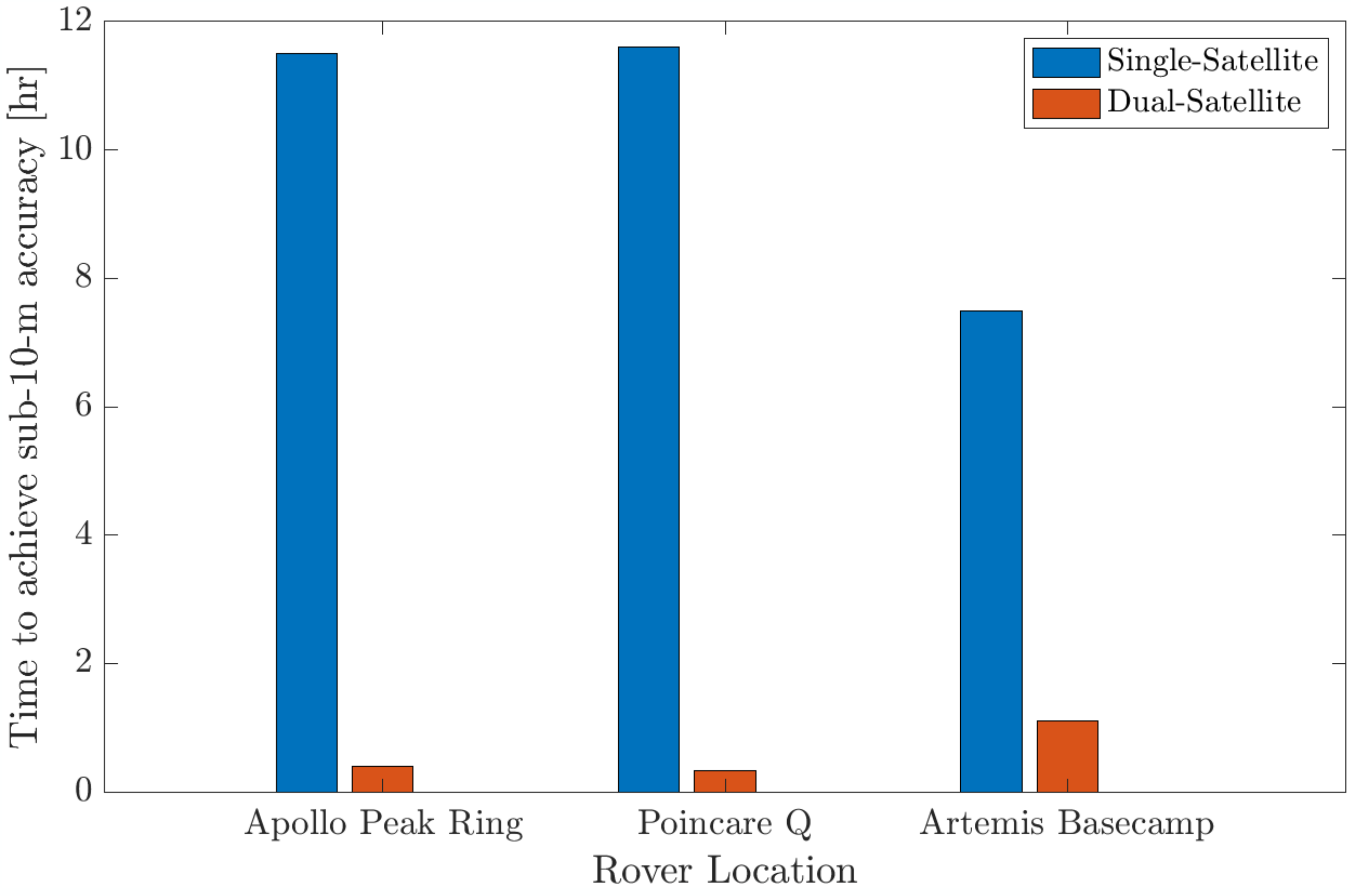
Key findings:

- Mean sub-10-m localization time is ~1 hour
- No overlapping occultation periods

Comparison to the Single-Satellite System

LCRNS ephemeris errors:
- σ_p = 4.48 m
- σ_v = 0.40 mm/s

97%, 97%, and 85% improvement with dual-satellite system in comparison to single-satellite system for each location, respectively.



Comparison to the Single-Satellite System

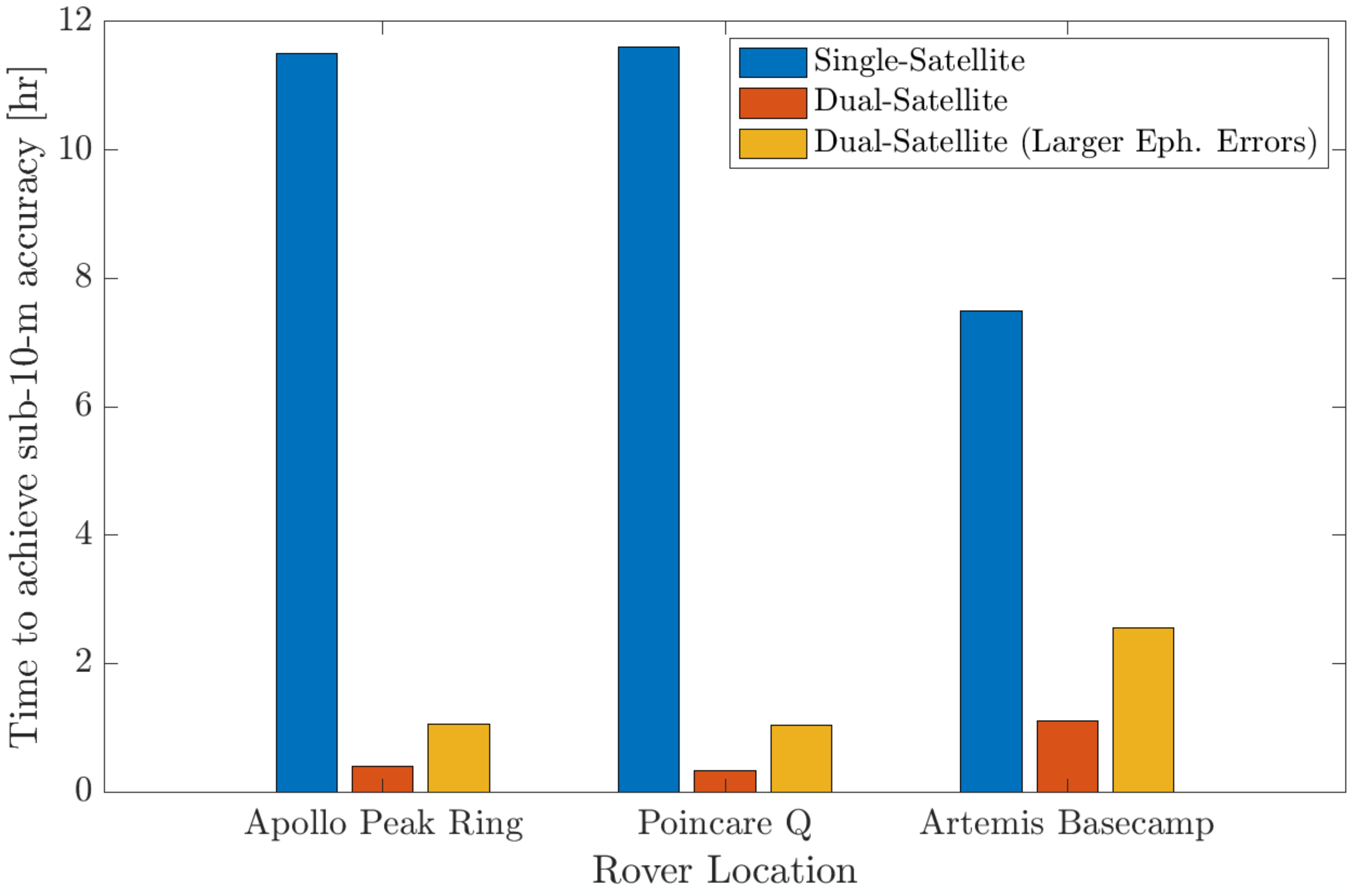
LCRNS ephemeris errors:

- σ_p = 4.48 m
- σ_v = 0.40 mm/s

Inflated ephemeris errors:

- σ_p = 100.0 m
- σ_v = 10.0 mm/s

91%, 91%, and 66% improvement with inflated eph. errors in comparison to single-satellite system for each location, respectively.



Concluding Remarks

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Key Questions

What will the orbit of the auxiliary satellite have to look like to best complement the Lunar Pathfinder's coverage?

What *improvement* in the rover's localization accuracy can be achieved by introducing a second satellite?

Main Results

We identified the optimal auxiliary satellite orbit that minimizes PDOP, maximizes visibility, and maintains an ELFO.

We achieved an 85% to 97% improvement over the single-satellite system when using similar ephemeris errors.

Future Work

- Perform 2D grid search on auxiliary satellite's orbital elements
- Fuse with measurements from the onboard sensors to localize the rover while it is in motion

Acknowledgements

Special thanks to the NAV Lab!

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State Estimation Framework

Weighted Batch Filter

Accumulate measurements.

$$\tilde{\mathbf{y}} = \left[\tilde{\dot{\rho}}_1(t_1) \quad \dots \quad \tilde{\dot{\rho}}_1(t_N) \quad \tilde{\dot{\rho}}_2(t_1) \quad \dots \quad \tilde{\dot{\rho}}_2(t_N)\right]^\top$$

Predict the next expected pseudorange rate measurements.

$$\hat{\dot{\rho}}_{k+1}(t) = \tilde{\mathbf{v}}_{\boldsymbol{s}}(t) \cdot \frac{\tilde{\mathbf{r}}_{\boldsymbol{s}}(t) - \hat{\mathbf{r}}_{\boldsymbol{r},k+1}}{\|\tilde{\mathbf{r}}_{\boldsymbol{s}}(t) - \hat{\mathbf{r}}_{\boldsymbol{r},k+1}\|} + c\left(\hat{\dot{\delta t}}_{r,k+1} - \hat{\dot{\delta t}}_s\right)$$

$$\hat{\mathbf{y}}_{k+1} = \left[\hat{\dot{\rho}}_{1,k+1}(t_1) \quad \dots \quad \hat{\dot{\rho}}_{2,k+1}(t_N)\right]^\top$$

Minimize cost function.

$$C = \|\tilde{\mathbf{y}} - \hat{\mathbf{y}}_{k+1}\|^2_{\mathbf{W}}$$

$$\delta\mathbf{x}_k = \left(\mathbf{J}_k^\top \mathbf{W} \mathbf{J}_k\right)^{-1} \mathbf{J}_k^\top \mathbf{W} \delta\mathbf{y}_k \quad , \quad \mathbf{W} = \mathrm{diag}\left(\boldsymbol{\sigma}^{-2}_{\boldsymbol{tot,1}} \ , \ \boldsymbol{\sigma}^{-2}_{\boldsymbol{tot,2}}\right)$$

Simulation Parameters	
Measurement sampling rate	1 Hz
Filter update interval	180 seconds
Number of Monte Carlo runs	100

Use weighted PDOP to determine measurement diversity.

$$\mathbf{M} = (\mathbf{J}_k^\top \mathbf{W} \mathbf{J}_k)^{-1}$$

$$PDOP_{\mathbf{W}} = \sqrt{M_{11} + M_{22} + M_{33}}$$

State Estimation Framework

- We can remove the initial spike by augmenting the measurement vector with the rover's initial position estimate
- Doing so does not compromise the long-term performance of the state estimation