Time-Correlated Ephemeris Errors and Relativistic Effects for Single-Satellite Doppler Localization of Lunar Rovers

Kaila M. Y. Coimbra and Grace Gao
Stanford University

In recent years, there is a strong renewed interest in developing a sustained presence on the Moon. A growing set of lunar mission architectures from space agencies and commercial providers are targeting rover operations supported by dedicated or hosted relay satellites (Keane et al., 2022; Fernando et al., 2025). In the early stages of lunar exploration, prior to full LunaNet constellations (Giordano et al., 2023), absolute rover localization with minimal infrastructure will be critical. In this work, we investigate the localization performance of a single lunar rover operating in the lunar southern region (an area of high scientific interest) using a single lunar satellite without a dedicated navigation payload. The rover is assumed to rely on only the Doppler shift of the satellite downlink signal, along with its onboard odometry measurements, to estimate its position over time.

RELATED WORKS AND LIMITATIONS

This work focuses on improving simulation fidelity and state estimation for single satellite Doppler-based localization, with an emphasis on satellite ephemeris uncertainty and relativistic effects. Pöhlmann et al. (2025) have incorporated temporally correlated error models in lunar navigation and argue that temporal correlations are frequently neglected in existing works. Their evaluation emphasizes hybrid architectures with more measurement diversity, including multiple satellites and a lunar reference station. Motivated by this gap, we focus on the single satellite case, where observability is limited and time correlation in the errors can dominate estimator behavior.

Recent works have also highlighted the need for well-defined lunar time systems, such as Lunar Coordinated Time (TCL), and for consistent conversions between Earth-based coordinate times and lunar time scales. Kopeikin and Kaplan (2024) and Bourgoin et al. (2026) provide formulations for these transformations. Iiyama and Gao (2026) incorporate relativistic modeling when performing orbit determination and time synchronization (ODTS) of a lunar satellite using weak GNSS signals. However, relativistic modeling is still commonly neglected in rover localization studies, even though deterministic relativistic rate terms can couple clock and orbital states in estimation. We address this limitation in the literature by incorporating relativistic corrections.

In prior work by the authors, we investigated single satellite rover localization using Doppler measurements, along with simplified noise models and an anchored batch estimation approach (Coimbra et al., 2025; Coimbra and Gao, 2025). In this work, we improve the fidelity of the simulated error sources and introduce a tightly coupled filtering and smoothing framework that can process these time varying effects.

PROPOSED WORK

In prior work, we added white noise to rover velocity to model odometry noise, modeled clock effects using a simplified constant bias, and simulated ephemeris uncertainty using white noise perturbations on satellite position and velocity. We also assumed an Earth based time reference and did not model TCL conversions or the corresponding deterministic relativistic rate terms. With higher fidelity modeling, these assumptions can lead to estimator inconsistency and can mask time correlated effects that are important in low observability single satellite regimes.

In this work, we improve simulation fidelity by modeling rover odometry as a noisy incremental displacement measurement, modeling both rover and satellite clocks using two state stochastic clock dynamics, and modeling satellite ephemeris errors as a time-correlated process with periodic ODTS updates. We also incorporate relativistic clock rate effects and light time delay in the measurement simulation and estimation framework. Given these time-varying error sources, we move from an anchored batch approach to a tightly coupled Extended Kalman Filter (EKF) with Rauch Tung Striebel (RTS) backward smoothing as a post-processing step.

PROBLEM SET UP

To enable direct comparisons to prior work, we employ a similar simulation setup and measurement model. We model the satellite using ESA SSTL Lunar Pathfinder specifications (SSTL, 2022). The satellite follows an elliptical lunar frozen orbit with an orbital period of 10.84 hours. We employ Lunar Pathfinder antenna parameters and assume the satellite antenna points in the nadir direction.

For rover signal availability, we apply an elevation mask of 5 degrees and a carrier-to-noise density ratio threshold of 30 dBHz, consistent with Nardin et al. (2023) and Melman et al. (2022). This yields an occultation duration of roughly 3 to 6 hours per 10.84 hour orbit depending on rover location. We model the rover high gain omnidirectional antenna using NASA Endurance rover specifications (Keane et al., 2022). We assume the rover initial position error is 5 m standard deviation (3D), motivated by human-in-the-loop map matching or stationary position refinement (Coimbra et al., 2025). The target performance objective is to converge to, or maintain, 10-m 3D position accuracy over time (Cauligi et al., 2023).

ERROR MODELING

For rover motion sensing, we assume onboard sensors such as inertial sensors, gyroscopes, LiDAR, and wheel encoders provide knowledge of incremental rover motion. We aggregate these effects into an odometry measurement model in which we add white noise to the incremental distance traversed between consecutive states. The noise standard deviation is proportional to the true distance traveled over the step.

For ground truth clock dynamics, the satellite and rover clocks independently follow a two state stochastic model that propagates clock bias and drift. The clock process noise is parameterized by q_1 and q_2 based on clock type. We assume the satellite carries a Rubidium Atomic Frequency Standard clock and the rover carries a lower grade chip scale atomic clock.

To model ground truth time-correlated ephemeris errors, we apply a first order Gauss Markov (FOGM) process on satellite acceleration. This is commonly referred to as a Singer model where acceleration is an exponentially correlated random process (Singer, 2007). To discretize the continuous time process noise for numerical evaluation, we use the Van Loan method for stability (Brown and Hwang, 1997). Under this model, the expected ephemeris error growth is unbounded without ODTS corrections. We therefore assume the satellite receives periodic ODTS updates from an Earth ground station or other lunar asset, and we model these corrections as resets at a fixed interval. Based on stated communication cadences in planned mission specifications (SSTL, 2022), we assume that the ODTS updates occur every 6 hours.

We also incorporate relativistic effects in the clock and measurement simulation. We propagate the rover clock in rover proper time, accounting for the gravitational potential at the surface. We propagate the satellite clock in satellite proper time while also accounting for the special relativistic velocity time dilation. We model light time delay between the rover and the satellite in the Doppler measurement calculations. We neglect Shapiro delay, which is a gravitational signal propagation delay that becomes significant for paths that pass close to massive bodies, since the rover-to-satellite link in this geometry yields a negligible contribution.

STATE ESTIMATION FRAMEWORK

In prior work, the rover state estimate included the rover's 3D position and a relative clock drift term. In this work, to account for biases induced by time-correlated ephemeris errors, we augment the rover state to include a scalar bias term. The EKF treats the satellite ephemeris as a known input, so unmodeled satellite ephemeris errors enter the Doppler observable as an effective time-correlated measurement bias. Modeling the full satellite error state would unnecessarily increase the estimator dimension and requires careful correlation modeling between orbital states and the measurement geometry. Instead, we model the dominant effect using a single bias state with FOGM dynamics. We assume the rover has knowledge of ODTS update times and we reduce the bias covariance at ODTS updates to reflect improved ephemeris knowledge after correction.

For rover dynamics, odometry is used as a control input to propagate the rover's position. Because Doppler measurements from a single satellite provide limited geometric observability, the filter can become overconfident in directions that are weakly constrained by the measurement. To prevent inconsistency, we inject additional process noise in the cross-track direction as a conservative model for unmodeled lateral effects, such as heading uncertainty and surface slip, and to maintain realistic covariance growth in weakly observable directions. We manually tune the cross-track noise and the steady-state bias standard deviation in the ephemeris bias process.

Relativistic clock rate effects introduce a deterministic rate contribution that must be separated from the stochastic oscillator behavior. We therefore represent the estimated relative drift as a deterministic component plus a stochastic component that is estimated by the filter. The Doppler measurement model is modified to incorporate the light time delay and to compute the relativistic coupling between orbital motion and clock states in a consistent time framework.

The EKF update processes Doppler measurements when the satellite is in view and refines the rover state throughout the simulation. We also apply a single pass of RTS backward smoothing (Rauch et al., 1965) to leverage future measurements and refine prior state estimates. In practice, this smoother is applied as post-processing to improve geotagging accuracy for samples collected along the rover trajectory.

SIMULATION VALIDATION

We validate the error models and estimation framework for rover locations at key waypoints in the lunar South Pole Aitken basin (Keane et al., 2022). We evaluate multiple rover motion profiles including constant speed traverses and trajectories with periodic stops for sampling or imaging. We conduct an ablation study on the relativistic modeling to quantify the impact of neglecting deterministic relativistic rate terms and light time delays on localization performance. We also study the degree of ephemeris error severity the estimator can tolerate before it can no longer recover to sub-10-m position accuracy over multiple satellite passes. Despite the low observability of the single-satellite Doppler system and the multiple time-varying error sources modeled for higher fidelity validation, we find that the tightly-coupled EKF with RTS smoothing is able to extract useful information from minimal measurements and maintain reasonable localization accuracy over time.

DESCRIPTION OF NEW AND INNOVATIVE ASPECTS

The new and innovative aspects of this work are as follows:

- **Effect of time-correlated ephemeris errors for a single satellite on rover localization:** To the best of the authors' knowledge, this work is the first to quantify how time-correlated ephemeris errors degrades single-satellite Doppler-only rover localization performance and to demonstrate mitigation using a tightly-coupled EKF with a correlated bias state and RTS smoothing.
- **Relativistic effects on lunar satellite-rover interaction:** To the best of the authors' knowledge, this work is the first to incorporate a consistent proper time and coordinate time treatment for both measurement simulation and estimation for rover localization using a single satellite and to quantify the localization impact when these effects are omitted.

REFERENCES

Bourgoin, A., Defraigne, P., and Meynadier, F. (2026). Lunar reference timescale. *Metrologia*, 63(1):015003.

Brown, R. G. and Hwang, P. Y. C. (1997). *Introduction to Random Signals and Applied Kalman Filtering*. John Wiley & Sons, Inc., 3 edition.

Cauligi, A., Swan, R. M., Ono, H., Daftry, S., Elliot, J., Matthies, L., and Atha, D. (2023). ShadowNav: Crater-Based Localization for Nighttime and Permanently Shadowed Region Lunar Navigation. *2023 IEEE Aerospace Conference*.

Coimbra, K. M., Cortinovis, M., Mina, T., and Gao, G. (2025). Single-satellite lunar navigation via Doppler shift observables for the NASA Endurance mission. *NAVIGATION: Journal of the Institute of Navigation*, 72(3).

Coimbra, K. M. and Gao, G. (2025). Single-Satellite Doppler-Based Localization for Lunar Rovers in Motion. In *Proceedings of the 38th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2025)*, pages 677–688.

Fernando, B., Neal, C., Kiraly, J., Fernandez, B., Patterson, R., Gyalay, S., and Lemelin, M. (2025). The Case for Continuing VIPER: A Critical Milestone on the Journey Back to the Moon. *The Planetary Science Journal*, 6(12):289.

Giordano, P., Swinden, R., Gramling, C., Crenshaw, J., and Ventura-Traveset, J. (2023). LunaNet Position, Navigation, and Timing Services and Signals, Enabling the Future of Lunar Exploration. In *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 3577–3588, Denver, Colorado.

Iiyama, K. and Gao, G. (2026). GNSS-based Lunar Orbit and Clock Estimation With Stochastic Cloning UD Filter. *arXiv preprint arXiv:2601.16393*.

Keane, J. T., Tikoo, S. M., and Elliot, J. (2022). Endurance: Lunar South Pole-Aitken Basin Traverse and Sample Return Rover. *NASA Jet Propulsion Laboratory, Technical Report*.

Kopeikin, S. M. and Kaplan, G. H. (2024). Lunar time in general relativity. *Physical Review D*, 110(8):084047.

Melman, F. T., Zoccarato, P., Orgel, C., Swinden, R., Giordano, P., and Ventura-Traveset, J. (2022). LCNS Positioning of a Lunar Surface Rover Using a DEM-Based Altitude Constraint. *Remote Sensing*, 14(16):3942.

Nardin, A., Minetto, A., Guzzi, S., Dovis, F., Konitzer, L., and Parker, J. J. K. (2023). Snapshot Tracking of GNSS Signals in Space: A Case Study at Lunar Distances. *Proceedings of the 36th International Technical Meeting of the Satellite Division of The Institute of Navigation (ION GNSS+ 2023)*, pages 3267–3281.

Pöhlmann, R., Staudinger, E., and Seco-Granados, G. (2025). Towards Hybrid Lunar PNT: Error Models, Lower Bounds and Algorithms. *arXiv preprint arXiv:2508.10699*.

Rauch, H. E., Tung, F., and Striebel, C. T. (1965). Maximum likelihood estimates of linear dynamic systems. *AIAA journal*, 3(8):1445–1450.

Singer, R. A. (2007). Estimating optimal tracking filter performance for manned maneuvering targets. *IEEE Transactions on Aerospace and electronic systems*, (4):473–483.

SSTL (2022). Lunar Pathfinder: Data relay satellite in orbit around the Moon. *SSTL, Service Guide (V4)*.